# All-Units Discounts as a Partial Foreclosure Device*

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September 26, 2017

**Abstract**

We investigate the strategic effects of all-units discounts (AUDs) used by a dominant firm in the presence of a capacity-constrained rival. Due to the limited capacity of the rival, the dominant firm has a captive portion of the buyer's demand for the single product. As compared to linear pricing, the dominant firm can use AUDs to go beyond its captive portion by tying its captive demand with part of the competitive demand and partially foreclose its small rival. When the rival's capacity level is well below relevant demand, AUDs reduce the buyer's surplus.

**Keyword:** all-units discounts, capacity, captive demand, partial foreclosure

**JEL code:** L13, L42

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*We thank the Editor, Benjamin Hermalin, and two anonymous referees for many insightful comments and suggestions, and we are grateful to many people for helpful discussions, especially Marcel Boyer, Luis Cabral, Juan Carrillo, Yongmin Chen, Zhijun Chen, Zhiqi Chen, Harrison Cheng, Joseph Farrell, Liliane Giardino-Karlinger, Ken Hendricks, Dmitry Lubensky, Massimo Motta, Daniel O'Brien, Cheng-Zhong Qin, Tom Ross, Michael Williams, Ralph Winter, and seminar participants at the University of Southern California, the University of Louisville, Koç University, Zhejiang University, Shandong University, the 2012 Southern California Symposium on Network Economics and Game Theory, the 9th Workshop on Industrial Organization and Management Strategy, the Annual Conference of Mannheim Centre for Competition and Innovation, the 11th Annual International Industrial Organization Conference, the 2013 North American Summer Meeting of the Econometric Society, the Workshop in Industrial Economics, Academia Sinica, Tsinghua University, CIRANO-PHELPS Session at the CEA 2014 Meetings, the 28th Summer Conference on Industrial Organization, Fall 2014 Midwest Economic Theory Conference, the 42nd EARIE Annual Meeting, 2016 Next Generation of Antitrust Scholars Conference, and Pennsylvania State University. The usual caveat applies.

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# 1 Introduction

A common form of pricing is all-units discounts (AUDs), in which the price per unit is cut on *all* units once the buyer's order crosses a threshold. AUDs and related conditional rebate schemes are frequently observed in intermediate-goods markets, and their adoption by dominant firms has become a prominent antitrust issue. For instance, in the *Tomra*[1] and *Michelin II*[2] cases, "individualised retroactive rebate schemes" used by Tomra, and quantity rebates used by Michelin, were found to be exclusionary. Additionally, the European Commission has found loyalty discounts adopted by dominant firms to be anticompetitive in several cases.[3]

All these cases are Section 2/Abuse of Dominance cases. By the very nature of dominance, part of the buyer's demand is captive to the dominant firm. In reality, such captive demand could arise from various sources: small rivals often have capacity constraints, as in the cases of *Tetra Pak,*[4] *Tomra*, *Michelin II* and *Intel*;[5] the dominant firm usually offers a must-carry brand to customers, as in the cases of *Post Danmark II*[6] *and Intel*. Regardless of where the captive demand comes from, the important fact is that the small rival can only compete for a portion of the buyer's demand. The major concern about an AUD scheme and its variations is their potential foreclosure effects on the competitive portion of the market. Intuitively, a dominant firm can take advantage of its captive portion of the demand to induce a buyer to purchase a significant portion of her requirements from it. Hence, AUDs can mean reduced sales for smaller rivals. Such reasoning has been employed in some of the cases discussed, as well as by the European Commission (see European Commission DG COMP Discussion Paper, 2005 and Guidance Paper, 2009). However, to the best of our knowledge, there has been no formal analysis of this claim in the economic literature.

In this article, we propose a model to formalize the idea that AUDs permit foreclosure when a dominant firm competes against a small rival. The dominant firm enjoys a captive demand due to the capacity constraints of its smaller rival. In particular, we consider a three-stage game with complete information, in

[1]Case COMP/E-1/38.113, C(2006)73, *Prokent-Tomra*, Commission Decision of 29 March 2006. Case T-155/06, *Tomra Systems and Others* v. *Commission*, Judgment of the General Court of 9 September 2010. Case C-549/10 P, *Tomra Systems and Others* v. *Commission*, Judgment of the Court (Third Chamber) of 19 April 2012.

[2]Case COMP/E-2/36.041/PO-Michelin, Commission Decision of 20 June 2001. Case T-203/01, *Manufacture Française des Pneumatiques Michelin* v. *Commission of the European Communities supported by Bandag Inc.*, Judgment of the Court of First Instance of 30 September 2003. See Motta (2009) for discussions of this case.

[3]To name a few, British Airways (Case C-95/04, *British Airways plc* v. *Commission of the European Communities supported by Virgin Atlantic Airways Ltd.*, Judgment of the European Court of Justice, March 2007), and Intel (Case COMP/C-3/37.990—Intel (2009); Docket No. 9341, In the Matter of Intel Corporation (2010)).

[4]On November 16, 2016, the State Administration of Industry and Commerce (SAIC) of China announced that, between 2009-2013, Tetra Pak abused its dominance in relevant product markets in China. One of the abusive practices cited in the SAIC's decision was a collection of discount policies conditional on volume thresholds.

[5]Intel (Case COMP/C-3/37.990—Intel (2009); Docket No. 9341, In the Matter of Intel Corporation (2010)).

[6]Case C-23/14, *Post Danmark II*.

which the dominant firm and its rival produce identical products with zero costs and make sequential price offers to a buyer before the buyer makes her purchase decision. We find that AUDs always increase the dominant firm's profits, sales, and market share over linear pricing (LP). At the same time, AUDs adopted by the dominant firm lead to partial foreclosure of the rival, in the sense that the rival's profits, sales, and market share are strictly reduced relative to what they would be under LP. These results hold for any capacity level of the small rival. When the rival's capacity level is especially low relative to demand, AUDs reduce the buyer's surplus but increase total surplus.[7]

The intuition for our findings is that, due to the limited capacity of the rival, the dominant firm is able to use AUDs to tie *part of* the competitive portion (as the tied good) to its captive portion (as the tying good) of a single product, leveraging its market power from the captive to the competitive demand. Under AUDs, the list price is set so high that it coerces the buyer into meeting the threshold if she buys anything from the dominant firm. The dominant firm always sets its quantity threshold above the captive demand size, together with a per-unit discount as an incentive. The discontinuity of AUDs forces the buyer to contemplate taking a "chunk" from the dominant firm, consisting of its captive portion and part of the competitive portion, rather than making a purchase decision on the marginal principle, as under LP. This effectuates a tie: the captive portion will not be sold unless the buyer purchases part of the competitive portion of the single product from the dominant firm.

Moreover, the optimal AUDs are a *partial* tying. It is *not* optimal for the dominant firm to tie the *whole* competitive demand to its captive demand, although it is able to do so. By leaving part of the competitive demand to the rival, the dominant firm can induce less aggressive responses from the rival than by fully excluding it, earning higher profits and hurting the rival at the same time. To ensure the leverage only affects part of, but not all of, the competitive demand, the dominant firm's pricing scheme must entail a quantity threshold encroaching only on part of competitive demand, above which and below which prices are different.

In AUDs, the quantity threshold and the corresponding discounted per-unit price act as a quasi-fixed fee with minimum quantity requirement. This feature leads to several effects on welfare. First, the quasi-fixed fee has a surplus extraction effect. Because the dominant firm can extract the buyer's surplus better under AUDs, AUDs can hurt the buyer. Second, AUDs have a quantity expansion effect. The quasi-fixed

[7] Chao and Tan (2013) have shown that even when the rival has a lower marginal cost than the dominant firm, the partial foreclosure results still hold, and AUDs may reduce total surplus.

fee enables the dominant firm to extract a surplus from the buyer more efficiently, and hence gives it more incentive to expand supply. Such a quantity expansion is carried out by a quantity threshold larger than the dominant firm's captive demand. That quantity threshold encroaches on the competitive demand, which hurts the small rival, but can increase total surplus and the buyer's surplus. Depending on the competitive pressure from the small rival's capacity level, the surplus extraction effect may dominate the quantity expansion effect and reduce the buyer's surplus, and vice versa. Third, when the small rival is more efficient, the quantity expansion from the dominant firm can have a negative effect on social welfare: because the dominant firm sells more than its captive demand and forces the more efficient rival to supply less, more output is supplied by the less efficient dominant firm under AUDs. Thus, the quantity expansion from the dominant firm may harm total surplus.[8]

The literature on AUDs is relatively new. Kolay, Shaffer, and Ordover (2004) show the price discrimination effect of AUDs offered by a monopolist when the downstream buyer has private information.[9] In a successive, bilateral monopolies setting, O'Brien (2017) shows that AUDs can facilitate non-contractible investments. Using detailed data from one retailer (a retail vending machine operator), Conlon and Mortimer (2015) study empirically the efficiency and foreclosure effects of AUDs used by a dominant chocolate candy manufacturer (Mars, Inc.) and find evidence that AUDs result in upstream foreclosure.

In the spirit of Aghion and Bolton (1987), Feess and Wohlschlegel (2010) show that AUDs can shift the rent from the entrant to the coalition between the incumbent and the buyer. Choné and Linnemer (2015) show that general nonlinear pricing can be exclusionary in the Aghion-Bolton framework with product differentiation. In Choné and Linnemer (2016), they introduce two dimensions of uncertainty into the Aghion-Bolton model with inelastic demand, and study the relationship between the shape of optimal nonlinear price-quantity schedule and the uncertainty. Ide, Montero, and Figueroa (henceforth, IMF) (2016) show that rebates without unconditional transfers in the Aghion-Bolton setting are not anticompetitive.

It is worth noting that there are several modeling differences between ours and IMF's. First, IMF, following Aghion and Bolton (1987), assume the small firm's cost is uncertain to the dominant firm and the buyer, whereas the small firm's cost in our model is certain and common knowledge to all. Second, IMF consider a horizontal demand consisting of a captive portion and a contestable portion with the same reservation value for both portions, whereas we consider a general downward-sloping demand curve so that

[8] Chao and Tan (2013) have shown this third effect.

[9] Wong (2014) compares the monopolist's profitability of AUDs with that of other pricing forms (e.g., incremental discounts) in a more general setting.

the reservation values for captive portion and contestable portion are different. Third, the small firm's LP is essentially a bulk price (and thus nonlinear pricing) in IMF, because the divisible demand in their model is horizontal. In our model with a downward-sloping demand, the small firm is restricted to LP. Indeed, in many antitrust cases, small firms only used simple LP.[10] All these modeling differences contribute to the differences in IMF's and our results.

More importantly, the underlying ideas in IMF (which is rent-shifting through unconditional payment) and in our article (which is partial foreclosure through tying) are different. The contract in IMF, like that in Aghion and Bolton (1987), serves to shift surplus from a more efficient entrant to the incumbent through the buyer. Therefore, the incumbent welcomes the more efficient entrant sometimes if the entrant's cost is uncertain so that inefficient exclusion can occur (and all the time if there is no uncertainty about the entrant's cost so that inefficient exclusion cannot occur). One of IMF's contributions is to show that such rent-shifting cannot work without unconditional payment from the buyer to the incumbent, and that with only unconditional payment, the incumbent would like to elicit purchase from and only up to captive demand. This explains why the pre-discounted price cannot be too high in IMF.

In contrast, in our model, the dominant firm partially forecloses the small firm by tying its captive demand with part of the contestable demand. For the tying, the dominant firm commits not to sell its captive demand unless the buyer purchases some contestable demand from the dominant firm. This explains why the quantity threshold in our model is beyond the captive portion but below the buyer's full demand, and the pre-discounted price is set high enough to goad the consumers into purchasing more than the captive portion even when there is entry by the small firm. For our partial foreclosure idea to work, the AUDs in our model does not need the unconditional payment as IMF identified as necessary for Aghion and Bolton's rent-shifting to work. Moreover, the entry deterrence is entirely driven by the coalition's uncertainty about the potential entrant, which sometimes causes exclusion of a more efficient entrant by mistake. In the Aghion-Bolton as well as IMF setting, if the incumbent and the buyer know the entrant's cost (as in our model), there will be no entry deterrence, because the incumbent and the buyer can form a coalition to fully extract all the efficiency gain from the entrant, by setting the liquidated damages contingent on the entrant's cost. Nevertheless, our partial foreclosure result clearly does not need uncertainty on the small firm's cost.[11] Thus,

[10]For instance, *Intel* and *LePage's* in the US, *Canada Pipe* in Canada, *Post Danmark II* and *Tomra* in Europe, and *Tetra Pak* in China.

[11]We thank an anonymous referee for suggesting this important discussion about our model and Aghion and Bolton (1987) and IMF (2016).

our exclusionary mechanism complements Aghion and Bolton (1987) for cases without uncertainty about potential entrants and without the purchasing commitment from the buyer.

There is a literature on exclusionary contracts with competition between asymmetric firms. Ordover and Shaffer (2013) consider exclusionary discounts in a two-period model, where one firm is financially constrained, and the buyer incurs switching costs after her first-period purchase. Our model differs because we consider a one-time purchase from the buyer, and thus there is no switching cost across periods. DeGraba (2013) demonstrates that the large firm can bribe downstream firms for exclusivity, provided that the size difference between the large firm and the small firm is sufficiently large. We consider a different model with no downstream competition and do not allow upstream firms to pay the buyer up-front for exclusivity. And we find that AUDs can have a partial foreclosure effect for any capacity difference between the large firm and small firm.

Another related literature studies market-share discounts, where discounts are conditional on a seller's percentage share of a buyer's total purchases, instead of an absolute quantity. Majumdar and Shaffer (2009) explain how market-share discounts can create countervailing incentives for a retailer with private information on demand. Inderst and Shaffer (2010) point out that the market-share discounts can dampen both intra- and inter-brand competition at the same time. Mills (2010) suggests the market-share discounts can induce non-contractible effort from retailers. Calzolari and Denicolo (2013) show that the market-share discounts can be anticompetitive when buyers have private information. Chen and Shaffer (2013) find that a less than 100% share requirement may be more effective in deterring entry than a 100% naked exclusionary contract in the model of Rasmusen, Ramseyer, and Wiley (1991) and Segal and Whinston (2000). As a complement to those mentioned above, our article suggests that we should place a cautious eye on the volume- or share-threshold based contracts when they are adopted by a dominant firm.

## 2 Model Setting

An AUD scheme is a triple $(p_o, Q, p_1)$ with $p_o > p_1$ and $Q > 0$. Here $p_o$ is the per-unit price when the quantity purchased is less than the quantity threshold $Q$, and $p_1$ is the per-unit price for *all* units once the quantity purchased reaches $Q$. In other words, the AUDs are pricing schemes that reward a buyer for purchasing some threshold quantity from a firm. In particular, the total payment for purchasing $q$ units under

AUDs is

$$T(q) = \begin{cases} p_o \cdot q & \text{if } q < Q \\ p_1 \cdot q & \text{if } q \geq Q \end{cases}.$$

There are two firms, say 1 and 2, in the upstream market that produce identical products with zero marginal cost. To capture a notion of dominance, we introduce capacity constraints for the small firm into the model. Specifically, firm 1 has capacity to serve the entire demand of the buyer, whereas firm 2 is capacity-constrained in the sense that it can produce only up to its capacity $k$.[12,13,14] In the downstream, we assume a representative buyer whose gross benefit from quantity $q$ is $u(q)$.

Assume the following sequence of play: Firm 1 (the dominant firm) makes public its pricing scheme, which could be an LP or AUD scheme; firm 2 (the minor firm) responds by making public its per-unit price; and, lastly, the buyer decides how many units to buy from each of the firms. As a necessary tie-breaking rule, when indifferent the buyer purchases from firm 2.

Note that there are three asymmetries in our treatment of the dominant and minor firms. First, firm 2 has a limited capacity $k$, which is our primary parameter for modelling its smallness. Second, firm 2 is limited to LP only. Third, firm 1 moves first.[15]

We assume the buyer's gross benefit function $u(\cdot)$ is increasing, strictly concave for any quantity, and twice differentiable below the welfare-maximizing quantity $q^e$, where $u'(q^e) = 0$ and $0 < q^e < \infty$, and $u'(0) > 0$. The quantity demanded by the buyer at per-unit price $p$ is thus $q(p) \equiv \arg\max_{q \geq 0}[u(q) - p \cdot q]$. The assumptions just made about $u(\cdot)$ ensure $q(p)$ exists and is uniquely determined by $u'(q) = p$ for $0 \leq p \leq u'(0)$. Let $V(p) \equiv u(q(p)) - p \cdot q(p)$ denote the buyer's surplus when she purchases optimally at per-unit price $p$.

Firm 2's capacity is strictly less than the socially efficient level of quantities; i.e., $0 < k < q^e$. Consequently, firm 2 cannot serve the buyer's entire demand when the two firms compete à la Bertrand. When firm 1 set price $p$ and the buyer gets her first $k$ units from firm 2, firm 1 faces a residual demand function.

[12]"Tomra's rivals, including those who had the potential to become strong competitors, were all small or very small companies, with a very low turnover and very few employees." (Paragraph 85, Case COMP/E-1/38.113, *Prokent-Tomra*, Commission Decision 2006).

[13]In *ZF Merito v. Eaton* case, "even if an OEM decided to forgo the rebates and purchase a significant portion of its requirement from another supplier, there would still have been a significant demand from truck buyers for Eaton product. Therefore, losing Eaton as a supplier was not an option." (D.C. No. 1-06-cv-00623).

[14]In the Intel case, it is widely known that AMD is capacity constrained, because large computer manufacturers have to carry a significant proportion of their CPU requirements from Intel.

[15]In an earlier version of this paper, we show that our results are robust to endogenizing both pricing options and timing of the game.

We consider this as firm 1's captive demand function, denoted as $q^{cap}(p) \equiv \max\{q(p) - k, 0\}$. Correspondingly, the competitive portion is $k$, for which both firms compete. Note that the efficient surplus from firm 1's captive portion is $S^{cap} \equiv V(0) - u(k)$.

Monopoly profit under linear pricing is $\pi(p) \equiv p \cdot q(p)$. To facilitate our analysis, we assume $\pi(\cdot)$ is concave. Let $p^m \equiv \arg\max_p \pi(p)$ denote the monopoly price and $q^m \equiv q(p^m)$ the monopoly quantity. In addition, let

$$\pi^R(Q) \equiv \max_p p \cdot [q(p) - Q], \text{ for } Q \in [0, q^e]$$

be the maximum profit based on the residual demand $q(p) - Q$. One can readily verify that $\pi^R(Q)$ is strictly decreasing and convex in $Q$. From the concavity of $\pi(p)$ and the fact that $\pi'(0) = q^e \geq Q$ and $\pi'(p^m) = 0 \leq Q$, it follows that there exists a unique $p^R(Q) \equiv \arg\max_p p \cdot [q(p) - Q] \in [0, p^m]$, implicitly given by

$$\pi'(p^R) = Q, \tag{1}$$

and $p^R(Q)$ is strictly decreasing in $Q$.

In the next three sections, we will determine the pure-strategy subgame perfect equilibrium (SPE) outcome of the sequential-move game, and study the properties of the equilibrium outcome.

# 3 Preliminary Analysis

In this section, we first establish our benchmark case: sequential LP vs LP. Then we use an example to illustrate how the AUDs adopted by the dominant firm can partially foreclose the minor firm by tying its captive portion to competitive portion.

□ **Linear pricing benchmark.** Here we derive the equilibrium when the dominant firm and the minor firm offer LP sequentially. This case will be our benchmark for comparisons when the dominant firm adopts the AUDs.

**Proposition 1 (Equilibrium of Sequential LP)** *In the linear pricing equilibrium, both firms offer $p^R(k) \in (0, p^m)$, where $p^R(\cdot)$ is given by (1). Firm 1 earns $\pi^R(k)$ with sales $q(p^R) - k$; firm 2 earns $p^R \cdot k$ with sales $k$; the buyer's surplus is $V(p^R)$.*

This proposition states that, when firm 1 is restricted to LP, it has to leave firm 2 its capacity $k$ and only focuses on its captive demand $q(p) - k$. The per-unit price from firm 1, which is available for the buyer's whole demand, forces firm 2 to undercut it, because otherwise firm 2 would have no sales. Once firm 2 undercuts, the buyer will consider firm 1's supply only after exhausting firm 2's capacity.

An immediate result following from Proposition 1 is the comparative statics below.

**Corollary 1** *For $k \in (0, q^e)$, as $k$ increases, the price offer $p^R(k)$ decreases, the buyer's surplus increases, and firm 1's profit decreases.*

As firm 2's capacity $k$ increases, competition becomes more intensive, from which the buyer benefits and firm 1 gets hurt. However, firm 2's profit is not necessarily monotonic in $k$, because there are two opposing effects on its price and sales, respectively: $p^R(k)$ falls whereas $k$ rises. Indeed, firm 2's profit increases with $k$ when $k$ is small, whereas it decreases with $k$ when $k$ is large.

□ **Intuition for the role of the AUDs.** Before the formal analysis, we provide a simple example to illustrate how the AUDs can help firm 1 to achieve higher profits, partially foreclose its rival and hurt the buyer, as compared to LP.

Consider a buyer's demand given by $q(p) = 10 - p$, which is generated by the gross benefit function $u(q) = (10 - q/2)q$. Assume two firms produce identical products with zero marginal cost. Firm 1 can serve at least 10 units, and firm 2 can produce at most $k = 2$ units.

Under LP, firm 2 undercuts firm 1's per-unit price and serves the first 2 units of the buyer's demand. As a result, firm 1 sets a monopoly per-unit price over the residual demand $q(p) - 2 = 8 - p$, which is $4$. In equilibrium, firm 1 sells $4$ units, firm 2 sells $2$ units. Firm 1 earns a profit of $16$, and firm 2 earns $8$. The buyer gets $18$.

Now, suppose firm 1 uses AUDs with a volume threshold $Q = 9$, $p_o = 10$, and $p_1 = 32.105/9 \approx 3.57$. Given the AUDs and $p_2$ from firm 2, the buyer has to choose between "meeting $Q$" and "not meeting $Q$." Meeting $Q$ means that the buyer will buy 9 units from firm 1 and the remaining $1 - p_2$ unit from firm 2, which results in buyer's surplus $BS^{DS} = u(10 - p_2) - 32.105/9 \cdot 9 - p_2 \cdot (1 - p_2) = 17.895 - p_2 + p_2^2/2$. Not meeting $Q$ implies that the buyer has to rely on firm 2 only, because it is not worth buying at $p_o = 10$ from firm 1, which leaves buyer's surplus $BS^{SS} = u(2) - 2p_2 = 18 - 2p_2$.[16] Clearly, the buyer will meet $Q$ if and only if $BS^{DS} = 17.895 - p_2 + p_2^2/2 \geq 18 - 2p_2 = BS^{SS}$, i.e., $p_2 \geq 0.1$. As a result, if firm 2 wants to sell at its full capacity $k = 2$, then it has to undercut below $0.1$. So the maximal profit it can achieve when

[16] Here DS is short for dual sourcing, and SS stands for single sourcing.

selling at 2 units is $0.1 \times 2 = 0.2$. Nevertheless, if firm 2, facing a residual demand $q(p_2) - Q = 1 - p_2$, optimally sets $p_2 = 0.5$, then it can earn $0.5 \times (1 - 0.5) = 0.25$, although it only sells $0.5$ unit, which is below its full capacity. So through the AUDs with $Q = 9$, a per-unit price $p_o = 10$ and discounted per-unit price $p_1 \approx 3.57$, firm 1 can induce the buyer to meet its quantity threshold $Q$, and it earns a profit of $32.105$, which exceeds what it earns under LP. Correspondingly, firm 2 sets its per-unit price $p_2^{AUD} = 0.5$ and earns a profit of $0.25$, which is lower than what it earns in the case of LP. On the other hand, the buyer gets $17.52$, which is lower than what she receives in the case of LP.[17]

This example illustrates that, as compared to LP, the AUDs partially foreclose the rival, lowering its profits, sales volume and market share. The buyer is harmed, too.

Interestingly, by adopting the AUDs, firm 1 achieves profits higher than *the maximum surplus from its captive demand* $S^{cap}$. Recall that firm 1's captive demand is $q^{cap}(p) \equiv \max\{q(p) - k, 0\}$, and the surplus from this portion is $V(p) - u(k) + pk$, whose maximum is $S^{cap} = V(0) - u(k)$. This can be implemented by a two-part tariff, with zero marginal price and fixed fee $S^{cap}$.[18]

Note that two-part tariff here is just one of the mechanisms that firm 1 can use to extract full surplus from its captive demand. *How do the AUDs enable firm 1 to go beyond its captive demand and do a better job in extracting more surplus than* $S^{cap}$*?* When firm 1 focuses on its captive demand, total pie is already maximized at $V(0)$ and the sum of firm 2's profit and buyer's surplus is $u(k)$. So if firm 1 wants to gain more profit than $S^{cap}$, then the sum of firm 2's profit and buyer's surplus must be strictly less than $u(k)$. Note that whenever firm 2 sells $k$ units to the buyer, their joint surplus must be at least $u(k)$. Consequently, in order to further increase its profit over $S^{cap}$, *it is necessary for firm 1 to encroach into the competitive portion and prevent firm 2 from selling at its full capacity* $k$.

To prevent firm 2 from selling its full capacity, firm 1 must induce the buyer to buy firm 2's product *only after* buying certain amount from firm 1, and commit to a minimum quantity requirement more than its captive portion so that the residual demand for firm 2 is less than $k$. For such a quantity requirement to be accepted by the buyer, *firm 1 must tie its captive portion to the competitive portion,* and design its pricing scheme in such a way that the buyer cannot afford to lose firm 1 as a supplier. Consequently, firm 2 now, instead of firm 1, becomes a supplier for the less-than-$k$ residual demand.

[17] This example illustrates one profitable deviation for firm 1 to use AUDs. The optimal AUDs and the equilibrium outcomes for the linear demand and general $k$ can be found in Section 5 as well as in Tables A1 and A2 in Appendix.

[18] In this linear demand example, under two-part tariff, firm 1 will set its per-unit price at marginal cost zero to maximize its captive demand and then use a fixed fee to extract all the surplus $S^{cap} = \int_2^{10} (10 - q)dq = 32$ from the last 8 units.

So the crux for the AUDs to work is its quantity threshold. Given that a buyer has no choice but to purchase some, although not all, of her requirement from the dominant firm, the dominant firm can set its quantity threshold above its captive portion and induce the buyer to accept a chunk of its products and thus less of its rival's. As a result, firm 2 is forced to undersupply and earns lower profits than when firm 1 chooses LP.

## 4 Equilibrium Analysis of the AUDs

In this section, we characterize the equilibrium when firm 1 offers the AUD scheme. We find that the AUDs always increase firm 1's profit and market share, and induce firm 2 to undersupply below its capacity level. There exists a threshold of capacity level below which both firm 2 and the buyer are worse off under AUDs than under LP.

We solve our sequential-move game by backward induction. It turns out that the determination of the leader's optimal AUDs can be reduced to a mechanism-design problem. In particular, by judiciously choosing a quantity threshold together with a payment structure, the leading firm induces the buyer to reach the threshold and firm 2 to be indifferent between supplying the residual demand at a higher price and being a sole supplier by undercutting (firm 2 and the buyer's outside option). Through this way, the leading firm can leverage its market power in its captive market to the competitive part for which the smaller firm would otherwise be interested in competing.

Below we will first present several lemmas, which offer a set of necessary conditions for equilibrium. The logic is supported by iterated elimination of dominated strategies using firm 1 and firm 2's forward thinking. We will then formulate firm 1's maximization problem and characterize the equilibrium.

□ **Buyer's problem: single-sourcing or dual-sourcing.** We begin with analyzing the buyer's purchase decisions in the last stage of the game.

Given the AUDs $(p_o, Q, p_1)$ offered by firm 1, and a uniform price $p_2$ from firm 2, the buyer's maximization problem

$$\max_{\substack{q_1 \\ q_2 \leq k}} [u(q_1 + q_2) - T(q_1) - p_2 \cdot q_2]$$

can be decomposed into the following two maximization problems. The first one is given by

$$\max_{\substack{q_1<Q\\ q_2\le k}}[u(q_1+q_2)-p_o\cdot q_1-p_2\cdot q_2], \tag{2}$$

which represents the case when the buyer does not meet firm 1's volume threshold $Q$. The second one is given by

$$\max_{\substack{\Delta\ge 0\\ q_2\le k}}[u(Q+\Delta+q_2)-p_1\cdot(Q+\Delta)-p_2\cdot q_2], \tag{3}$$

which represents the case when the buyer meets firm 1's volume threshold $Q$. The buyer chooses one of the two options that gives her higher surplus.

*Single Sourcing from Firm 2.* In order for the AUDs to improve firm 1's profit over LP, the buyer must meet firm 1's volume threshold $Q$ in the AUD equilibrium. This is because the outcome of (2) can always be achieved by LP ($p_o$) vs LP ($p_2$). Therefore, firm 1 does not want the buyer to choose (2) in equilibrium, and it is without loss of generality to restrict our attention to $p_o=\infty$.[19] In what follows, we use $(Q,p_1)$ to denote the AUD scheme.

As a result of sufficiently high $p_o$, (2) is reduced to

$$\max_{q_2\le k}[u(q_2)-p_2\cdot q_2], \tag{SS}$$

which represents *single-sourcing* (SS) when the buyer does not meet firm 1's volume threshold and thus purchases from firm 2 only.[20] That is, under AUDs, if the buyer decides not to meet $Q$, she essentially chooses SS from firm 2.

The solution to (SS) problem serves as an outside option for firm 2 as well as for the buyer. Denote the buyer's demand under SS as $\overline{q}(k,p_2)\equiv\min\{k,q(p_2)\}$. We can write the buyer's surplus under SS as

$$BS_S(p_2)=u(\overline{q}(k,p_2))-p_2\cdot\overline{q}(k,p_2). \tag{4}$$

[19]Here $p_o$ does not have to be $\infty$, literally. In fact, we only need $p_o$ to be above a certain level in equilibrium, ensuring that any amount below $Q$ from firm 1 is never optimal for the buyer.

[20]Note that there is another kind of SS in which the buyer only purchases from firm 1. However, as will be shown in the proof of Lemma 1, introducing the buyer SS from firm 1 can at most give firm 1 $S^{cap}$.

The two firms' profits under SS are $\pi_1 = 0$ and

$$\pi_2 = p_2 \cdot \overline{q}(k, p_2). \tag{5}$$

*Dual Sourcing.* Now we study (3) carefully, as this is the case that emerges in equilibrium.

Under (3), after the buyer meets firm 1's volume threshold, she continues to buy from the cheaper source, as long as her marginal benefit is above the corresponding price. Thus, in order to have positive sales, firm 2 as a follower must always set $p_2 \le w \equiv \min\{p_1, u'(Q)\}$ as long as $0 < w$. As a result, the buyer buys exactly $Q$ units from firm 1 and her residual demand from firm 2. With $p_2 \le w$, (3) will be reduced to

$$\max_{q_2 \le k} [u(Q + q_2) - p_1 \cdot Q - p_2 \cdot q_2], \tag{DS}$$

which represents *dual-sourcing* (DS) when the buyer meets firm 1's volume threshold and continues to purchase her remaining demand from firm 2.

Under DS, firm 1 would never allow the buyer the freedom to purchase $k$ units from firm 2 without interfering with meeting its $Q$ requirement, when firm 2 simply matches firm 1's price $p_1$. That is, we cannot have $p_1 \le u'(Q + k)$, because $q(p_1) \ge Q + k$ and $p_2 \le w$ together imply that the buyer can meet $Q$ even after purchasing $k$ units from firm 2 first, which cannot be a profitable improvement over $S^{cap}$ for firm 1. Hence, we must have $u'(Q + k) < p_1$, and it follows that $u'(Q + k) < w$.

Because $u'(Q+k) < w$, the buyer's purchase when $p_2 \le w$ will be $\overline{q}(Q+k, p_2) = \min\{Q+k, q(p_2)\}$. So the buyer's surplus in (3) is

$$BS_D(p_2) = \begin{cases} u(\overline{q}(Q + k, p_2)) - p_2 \cdot \overline{q}(Q + k, p_2) + (p_2 - p_1) \cdot Q & \text{if } p_2 \le w \\ u(q(w)) - p_1 \cdot q(w) & \text{if } w < p_2 \end{cases}. \tag{6}$$

The two firms' profits from (3) are

$$\pi_1 = \begin{cases} p_1 \cdot Q & \text{if } p_2 \le w \\ p_1 \cdot q(w) & \text{if } w < p_2 \end{cases}, \tag{7}$$

and

$$\pi_2 = p_2 \cdot [\overline{q}(Q + k, p_2) - Q] \tag{8}$$

for $p_2 \leq w$, and zero otherwise.

*Single Sourcing or Dual Sourcing?* As firm 1 would have no sales under SS, in order for firm 1 to earn positive profit, it must ensure the buyer chooses DS under an AUD scheme. The following lemma shows that the buyer will meet firm 1's quantity threshold $Q$ in the AUD equilibrium, and firm 2 will supply too, but at a level strictly below its capacity $k$.

**Lemma 1 (Firm 1 must induce DS and firm 2 undersupplies)** *In the all-units discount equilibrium, (i)* $q_1 = Q \in (0, q^e)$*; (ii)* $0 < q(p_2) - Q < k$*.*

Observe the buyer buys from both firms: $Q$ from firm 1 and $q(p_2) - Q$ from firm 2. So firm 2 is the residual supplier after $Q$. Note that after the buyer fulfills firm 1's threshold $Q$, firm 2 will always set $p_2 < u'(Q)$, because otherwise the buyer would never buy anything from firm 2 in DS. So $Q < q(p_2)$ indicates that firm 1 will leave some demand for firm 2 under AUDs. But at the same time firm 1 constrains firm 2. $q(p_2) - Q < k$ implies that in the AUD equilibrium, firm 2 strictly undersupplies as a residual demand supplier. This contrasts with the case of LP, where firm 2 supplies its full capacity.

We now discuss two price constraints imposed by the equilibrium AUD. First, the buyer's option to purchase incremental units at $p_1$ means firm 2 faces the additional constraint $p_2 \leq p_1$. Second, in the AUD equilibrium, $p_1$ cannot be set too high: i.e., $p_1 \ngeq u'(k)$, because otherwise the buyer always chooses SS when $p_2 \leq p_1$. They are highlighted in the lemma below.

**Lemma 2 (Price Constraints Under AUD)** *The equilibrium all-units discounts* $(Q, p_1)$ *need to satisfy the following two constraints:*

$$p_1 < u'(k), \tag{C1}$$

*and*

$$p_2 \leq p_1. \tag{C2}$$

□ **Firm 2's Implied Threat Price.** From (4) and (6), the buyer's surplus curves under both SS and DS weakly decrease with $p_2$, and $BS_S$ curve as a function of $p_2$ is everywhere no flatter than $BS_D$ curve. Intuitively, the impact of $p_2$ on $BS_S$ is larger than that on $BS_D$, because firm 2 is the sole supplier under SS whereas firm 1, as a substitute supplier, becomes available under DS.

If $BS_D$ is everywhere below $BS_S$, then the buyer would never choose DS. But if $BS_D$ is everywhere above $BS_S$, it is not optimal for firm 1, either. Note that $BS_D$ decreases with $p_1 \cdot Q$. Whenever $BS_D$

is everywhere above $BS_S$, although the buyer will choose DS, firm 1 can always increase its profit by increasing $p_1 \cdot Q$. Hence, $BS_D$ and $BS_S$ must cross once, as shown in Figure 1. Such a unique crossing point is firm 2's threat price to undercut and induce SS.

[INSERT FIGURE 1 HERE]

**Lemma 3 (Firm 2's equilibrium threat price)** *In the all-units discount equilibrium, there exists a unique* $x \in (u'(Q+k), w)$ *determined by*

$$u(k) - x \cdot k = V(x) + (x - p_1) \cdot Q, \tag{9}$$

*such that* $BS_S(p_2) \gtreqless BS_D(p_2), \forall p_2 \lesseqgtr x$.

The left-hand side (LHS) of (9) is $BS_S$ at $p_2 = x$ when buying $k$ from firm 2 only. The right-hand side (RHS) of (9) is $BS_D$ at $p_2 = x$ when buying $Q$ from firm 1 and residual demand $q(x) - Q$ from firm 2. The condition (9) uniquely determines such $x$ at which the buyer is indifferent between SS and DS, given $(Q, p_1)$.

Given the AUDs $(Q, p_1)$ from firm 1, firm 2 can always induce the buyer to choose SS by undercutting sufficiently. The upper bound of such an undercutting threshold for SS is threat price $x$. That is, if firm 2 charges a price below $x$, the buyer will choose SS from firm 2 only for $k$. If firm 2's price is above $x$, the buyer will choose DS.

Now we can see firm 2's trade-offs introduced by the AUDs. Such trade-offs are absent under LP. Under LP, firm 2's only viable option is to undercut or match firm 1's per-unit price $p_1$, as $p_1$ is uniformly applied to all units supplied by firm 1. Nonetheless, with the quantity requirement $Q$, firm 1 commits to supply only $Q$ units with a payment $p_1 \cdot Q$ as long as $p_2 \leq w$, and thus creates trade-offs for firm 2: undercuts below $x$ to be a monopoly supplier, or instead charges a price above $x$ to supply the residual demand beyond $Q$. So *the most firm 1 can extract using $p_1 \cdot Q$ is the incremental surplus the buyer and firm 1 as a coalition can gain over the buyer's outside option of SS from firm 2 only, when firm 2 undercuts at $x$.* From (9), the total payment $p_1 \cdot Q$ to firm 1 is determined as

$$p_1 \cdot Q = V(x) + x \cdot Q - [u(k) - x \cdot k]. \tag{10}$$

□ **Firm 2's Pricing Decision.** Lemma 3 tells us that, if firm 2 sets its $p_2$ below the cutoff $x$, then it will be a monopoly supplier for $k$; if it sets its $p_2$ above $x$ but below $w$, then it will supply the residual demand $q(p_2) - Q$. As a result, firm 2's profit can be written as

,

$$\pi_2(p_2) = \begin{cases} p_2 \cdot k & \text{if } p_2 < x \\ p_2 \cdot [q(p_2) - Q] & \text{if } x \leq p_2 \leq w \\ 0 & \text{if } w < p_2 \end{cases} .$$

Note that there is a discontinuous drop at $x$ in firm 2's profit curve, which is shown as the red curves in Figure 2.

[INSERT FIGURE 2 HERE]

From its profit curve, we can clearly see the trade-offs firm 2 faces: undercutting below $x$ with its limited capacity $k$ and making itself a monopoly supplier, or giving up part of the competitive portion by leaving $Q$ units to firm 1 but charging a higher price between $x$ and $w$. Accordingly, firm 1's profit is

$$\pi_1 = \begin{cases} 0 & \text{if } p_2 < x \\ p_1 \cdot Q & \text{if } x \leq p_2 \leq w \\ p_1 \cdot q(w) & \text{if } w < p_2 \end{cases} .$$

Note that firm 2 would never choose $p_2 > w$, because it would earn zero in that case. But setting $p_2 < x$ would leave zero profit for firm 1. Thus, for a profitable improvement, firm 1 must ensure $x \leq p_2 \leq w$, instead of $p_2 < x$. That is,

$$\max_{p_2 < x} p_2 \cdot k = x \cdot k \leq \max_{x \leq p_2 \leq w} p_2 \cdot [q(p_2) - Q], \tag{11}$$

which says being a residual demand supplier is at least as profitable as being an undercutting monopoly. Because there is a discontinuous drop at $x$ in firm 2's profit curve, firm 2 would prefer $p_2 < x$ if $p_2 = x$ is the optimal solution to the RHS problem in (11). Thus, firm 2's optimal price $p_2$ must be an interior solution. We can further show that the inequality (11) must be binding in equilibrium.

**Lemma 4 (Firm 2's Choices)** *In the all-units discount equilibrium,*

$$x \cdot k = \pi^R(Q), \tag{12}$$

*and* $p_2 = p^R(Q) \in (x, w]$*, that is,*

$$\pi'(p_2) = Q. \tag{13}$$

The LHS of (12) is firm 2's profit when it supplies $k$ as an undercutting monopoly. The RHS of (12) is firm 2's maximum profit when it supplies the residual demand and undersupplies. Recall from (10) that $p_1 \cdot Q$ increases with $x$, as $u'(Q+k) < x$. So whenever the LHS of (12) is smaller than the RHS of (12), firm 1 can always increase its profit by increasing $p_1 \cdot Q$, thereby increasing threat price $x$. Lemma 4 demonstrates that in equilibrium, firm 1 will design its AUDs to induce firm 2 to be just satisfied as a residual demand supplier, rather than an undercutting sole supplier. In the AUD equilibrium, firm 2 undersupplies and sets its price $p_2$ above threat price $x$ to maximize the residual profit.

□ **Firm 1's Optimal AUDs.** Note that firm 1's choice of the AUD scheme can be reduced to an incentive contract design problem in which firm 1 chooses $(Q, p_1)$ to maximize its profit such that (i) the buyer prefers DS to SS, and (ii) firm 2 chooses its uniform price $p_2$ optimally and yet is indifferent between undersupplying at a higher price and selling its full capacity at a lower price. From the above discussion, firm 1's optimization problem is

$$\max_{(Q,p_1)} \pi_1^{AUD} = p_1 \cdot Q \tag{OP-AUD}$$

$$s.t. \text{ (9), (12), (13)}$$

$$\text{(C1), (C2)}$$

$$u'(Q+k) < x < p_2 < u'(Q) \tag{14}$$

To better understand strategic roles of the quantity threshold, we now denote all variables in terms of $Q$. For $0 \le Q \le q^e$, let $x(Q)$ satisfies (12). Using (10) (or (9)), the profit function of firm 1 can be expressed as

$$\pi_1^{AUD}(Q) = \underbrace{V(x) + x \cdot Q}_{\text{Sum of surpluses for firm 1 and the buyer under DS at } x} - \underbrace{[u(k) - x \cdot k]}_{BS \text{ under SS at } x},$$

where $x = x(Q)$ is determined by (12). From this profit expression, in the AUD equilibrium, firm 1 extracts all the incremental surplus over the buyer's outside option at threat price $x$. Note that when $x = 0$, the profit above is $S^{cap} = V(0) - u(k)$. As will be shown later, $x = 0$ satisfies all equality constraints, except for (14), and in the AUD equilibrium, (14) is never binding. So the AUDs can at least reach $S^{cap}$ by choosing $Q = q^e$.

Note that

$$\begin{aligned}\frac{d\pi_1^{AUD}}{dQ} &= \frac{\partial \pi_1^{AUD}}{\partial Q} + \frac{\partial \pi_1^{AUD}}{\partial x} \cdot x'(Q) \\ &= \underbrace{x}_{\text{Direct Effect}} + \underbrace{\{k - [q(x) - Q]\} \cdot x'(Q)}_{\text{Indirect Effect}}.\end{aligned} \tag{15}$$

Clearly, when $Q$ increases by one unit, firm 1 has to incur an extra zero per-unit production cost whereas it saves $x$, because $x$ is the amount of per-unit payment to firm 2 for a coalition of firm 1 and the buyer. So $x$ is thus the direct effect of setting a higher $Q$. There is an indirect effect of increasing $Q$. It is through its impact on the most profitable undercutting price $x(Q)$. Recall that from (10), *the most firm 1 can extract using $p_1 \cdot Q$ is the incremental surplus the buyer and firm 1 as a coalition can gain over the buyer's outside option of SS from firm 2 only, when firm 2 undercuts at $x$.* By the Envelope theorem, an increase in $x$ reduces $BS$ under SS by $k$. This helps firm 1, as it needs to compensate the buyer less when inducing DS. Meanwhile, the higher $x$ means the sum of surpluses for firm 1 and the buyer under DS is reduced, thanks to the greater payment to firm 2. By the Envelope Theorem, the magnitude of such reduction in surplus (or the increased payment to firm 2) is the residual demand purchased from firm 2 under DS at $x$, i.e., $q(x) - Q$. This hurts firm 1's profit. Consequently, the overall impact from $x$ is $k - [q(x) - Q]$. So the indirect effect of $Q$ through $x$ is $\{k - [q(x) - Q]\} \cdot x'(Q)$. To maximize its profit, firm 1 will balance these two effects, taking into account inequality constraints (C1), (C2) and (14).

From (12), we get $x = \pi^R(Q)/k$ and $x'(Q) = \pi^{R\prime}(Q)/k = -p_2/k$. Substituting these into (15) yields

$$\frac{d\pi_1^{AUD}}{dQ} = \frac{p_2}{k} \cdot \{[q(p_2) - Q] - [k - (q(x) - Q)]\}.$$

So (15) becomes

$$q(p_2) - Q = k + Q - q(x). \tag{FOC}$$

That is, firm 1 sets its volume threshold to balance the direct effect measured by the residual demand $q(p_2) -$

$Q$ and the indirect effect measured by the difference $k - [q(x) - Q]$.

To ensure the sufficiency and the uniqueness of (FOC) for the optimum and facilitate our comparative statics analysis, we assume $q''(p) \leq 0, \forall p \in [0, u'(0)]$, which we maintain in the rest of the article. The concave demand guarantees that $\pi_1^{AUD}(Q)$ is single-peaked in $Q$, and thus (FOC) characterizes the optimal solution. It is satisfied by generalized linear demand such as $q(p) = 1 - p^r$ $(r \geq 1)$. The following proposition summarizes our equilibrium analyses.

**Proposition 2 (AUD Equilibrium)** *The all-units discount equilibrium exists with $p_o = \infty$ and is characterized as follows. There exists a unique $\widehat{k} \in (0, q^e)$ such that*

- *when $k \in (0, \widehat{k})$, the equilibrium outcome $(Q, p_1, p_2)$ along with threat price $x$ is jointly determined by (9), (12), (13), and (FOC);*
- *when $k \in [\widehat{k}, q^e)$, the equilibrium outcome $(Q, p_1, p_2)$ along with threat price $x$ is jointly determined by (9), (12), (13), and the binding (C2).*

We now provide further intuition for how the AUDs work in equilibrium. As we discussed before, under LP, firm 2 always undercuts and sells at its full capacity. So the competitive portion $k$ becomes firm 2's turf. Accordingly, the best firm 1 can do is to extract the incremental surplus from its captive demand. Such incremental surplus is maximized at the efficient outcome, and thus firm 1 extracts its marginal contribution to the efficiency $S^{cap}$. How can the AUDs further increase firm 1's profit over $S^{cap}$, given that the outcome is efficient and firm 1 has already extracted the full surplus from its captive portion $q^e - k$? *The crux is to leverage its market power from the captive portion to the competitive portion, and at the same time prevent firm 2 from undercutting*.

The unique component of the AUDs, compared with LP, is the quantity requirement $Q$. Under AUDs, firm 1 now can take the initiative to dictate a quantity target beyond its captive portion, and commit not to supply any amount other than that. By doing so, the buyer faces trade-offs between SS and DS—if she buys from firm 2 at $p_2$ for $k$, she would not be able to meet firm 1's quantity requirement, and thus is forced to rely on firm 2's limited supply only; instead, if she meets firm 1's quantity target, her residual demand does not allow her to enjoy firm 2's lower price up to firm 2's full capacity. So with the quantity target instrument, firm 1 acts more aggressively and encroaches on the competitive portion. It induces the buyer to treat firm 2, instead of firm 1, as a residual demand supplier.

Correspondingly, under an AUD scheme, firm 2 now faces trade-offs that are missing under LP. Recall that under LP, firm 2's only option to survive is to undercut and hence sell its full capacity. In contrast, with AUDs, firm 2 has two options—undercut low enough to be a sole supplier, or set a high price serving the residual demand only. Hence, the quantity target creates another option other than undercutting for firm 2, so that preventing undercutting that is implausible under LP becomes possible now.

Recall from Lemma 2 that the AUDs need to satisfy two price constraints (C1) and (C2). We find that the former one is never binding, whereas the latter one $p_2 \leq p_1$ might be binding, depending upon $k$. When $k$ is small, firm 1 can extract surplus without worrying too much about competition. It will set a large requirement $Q$, and its average price for the $Q$ units $p_1$ will be high, too. From (13), the large $Q$ squeezes firm 2's residual demand and forces its optimal price $p_2$ to be low. So (C2) is not binding in this case. On the contrary, when $k$ is large, the market becomes more competitive as firm 2's capacity grows. The competitive pressure forces firm 1 to set a small $Q$ as well as a low average price for the $Q$ units. The small $Q$ results in a high $p_2$ from (13). That is, as $k$ increases, $p_1$ is forced to fall whereas firm 2's optimal price rises. Then the constraint (C2) becomes binding and, in equilibrium, firm 2 will just match $p_1$ by setting $p_2 = p_1$. So the equilibrium condition (FOC) is replaced with (C2) when $k$ is large.

□ **Properties of the AUD Equilibrium.** The corollary below illustrates the quantity expansion effect of AUDs.

**Corollary 2 (Quantity Expansion of the AUDs)** *In the all-units discount equilibrium, $Q > q^e - k$ for any $k \in (0, q^e)$.*

Under AUDs, firm 1 will expand its quantity requirement so much that the buyer would not be able to absorb firm 2's full capacity, even if firm 2 undercuts towards zero marginal cost. Note that $Q > q^e - k > q(p_2) - k$ for any $p_2 > 0$. So Corollary 2 is stronger than Part (ii) of Lemma 1. Such a significant quantity expansion squeezes the buyer's demand for firm 2's product to a level that it is strictly below its full capacity for any above-cost price it can charge. This illustrates how the dominant firm can leverage its market power from its captive portion to the competitive portion of the demand. This leverage is realized through what is effectively a refusal-to-deal threat if the buyer's purchase is less than the threshold, like a tying requirement: the sale of captive demand (the tying good) is conditional on the purchase of some competitive demand (the tied good).

Define the total surplus $TS$ as the sum of both firms' profits and the buyer's surplus. The following corollary summarizes how the equilibrium outcomes change as $k$ varies, when (C2) is not binding.

**Corollary 3 (The Impacts of Limited Capacity)** *For $k \in (0, \widehat{k}]$, as $k$ increases, the followings hold:*

*(i) the equilibrium quantity threshold $Q$ and the total output decrease;*

*(ii) the equilibrium $p_2$ (and also $x$) increases;*

*(iii) the equilibrium profit $\pi_1^{AUD}$ decreases, and $\pi_2^{AUD}$ increases;*

*(iv) $TS^{AUD}$ decreases.*

As $k$ increases, firm 2's competitive position becomes stronger. Therefore, firm 1, when designing its quantity target, has to leave more room for firm 2, in order to prevent firm 2 from undercutting. So the equilibrium $Q$ decreases as $k$ increases. Other comparative statics follow from the pattern of $Q$. The results that $\pi_1^{AUD}$ decreases whereas $\pi_2^{AUD}$ increases when $k$ increases are easy to understand: if we allow firm 2 to change its capacity level, it has a strong incentive to increase its capacity when $k$ is small. This is due to two effects: one, a larger capacity implies it can sell more in equilibrium; the other is the strategic effect on firm 1's equilibrium AUD design. This is in contrast with models in which the incumbent uses idle capacity to deter entry: here, instead, it is the small firm that gains a competitive advantage from its idle capacity.[21]

When $k$ is above $\widehat{k}$, (C2) binds, so there are no comparative statics for $k$ in that range. Given that most antitrust cases involving AUDs are abuse of dominance ones, by definition of dominance, it is natural to focus on the small $k$ case, in which, as observed, firm 2 does have incentives to expand capacity.[22] In Section 5, we use an example with linear demand to illustrate the comparative statics of AUDs for a full range of values of $k$.

# 5 A Comparison between AUDs and LP

In this section, we provide a comparison of LP and AUD equilibria, and demonstrate the impacts of AUDs.

Note that the LP equilibrium price $p^R$ decreases with $k$, whereas the AUD equilibrium price $p_2^{AUD}$ increases with $k$ as long as (C2) is not binding. Because $p_2^{AUD}(0) = 0 < p^R(0) = p^m$, there must be a cutoff $k_0 > 0$ such that $p_2^{AUD}(k_0) = p^R(k_0)$.

**Proposition 3 (Comparison with LP)** *(i) Prices: $p_2^{AUD} < p_2^{LP}$, for any $k \in (0, \min\{k_0, \widehat{k}\})$;*

[21]We thank Joseph Farrell for pointing out this to us.
[22]We thank the Editor and an anonymous referee for this helpful suggestion.

*(ii) Quantities:* $q_1^{LP} < q_1^{AUD}, q_2^{AUD} < q_2^{LP} = k$*, for any* $k \in (0, q^e)$;

*(iii) Profits:* $\pi_1^{LP} < \pi_1^{AUD}, \forall 0 < k < q^e; \pi_2^{AUD} < \pi_2^{LP}$*, for any* $k \in (0, \min\{k_0, \widehat{k}\})$;

*(iv) Buyer's Surpluses: there exists a* $k_1 \in (0, q^e)$ *such that* $BS^{AUD} < BS^{LP}$ *for any* $k \in (0, k_1)$*;*

*(v) Total Surpluses: there exists a* $k_2 \in (0, q^e]$ *such that* $TS^{AUD} > TS^{LP}$ *for any* $k \in (0, k_2)$.

When $k$ is relatively small, firm 1 gains from AUDs; firm 2 loses in terms of profit, sales, and market shares; and the buyer gets hurt, relative to an LP equilibrium. In the limiting case when $k$ goes to zero, the equilibrium price under LP is close to the monopoly price because the rival does not put any competitive pressure on the dominant firm. On the other hand, under AUDs, the equilibrium quantity threshold is close to the welfare-maximizing quantity level $q^e$ and the dominant firm can almost extract all the surplus from the buyer. (This, admittedly, is the usual difference between LP and what is, effectively, perfect price discrimination.) As a result, the buyer is worse off under AUDs than under LP when $k$ is small. In the following examples, we find that $k$ does not have to be really small in order for the results in Proposition 3 to hold. So under AUDs, we have partial foreclosure in the sense that firm 2 under-supplies strictly below its capacity and its profit is reduced. If firm 2 has a fixed cost of operation, then the AUDs adopted by a dominant firm can, by reducing firm 2's profit, induce firm 2 to exit. Our results support the antitrust concern on AUDs when $k$ is relatively small.

Compared with LP, AUDs have a quasi-fixed fee $p_1 Q$ at the quantity threshold $Q$. Such a quasi-fixed fee has two effects: a quantity-expansion effect and a surplus-extraction effect. On the one hand, because firm 1 can extract incremental surplus using this quasi-fixed fee, it has an incentive to push the equilibrium outcome towards a more efficient one. Such a quantity expansion effect tends to increase total surplus and buyer's surplus. On the other hand, because of the very quasi-fixed fee, firm 1 can extract surplus from the buyer more efficiently. Such a surplus extraction effect reduces buyer's surplus. Of course, firm 1's surplus extraction is constrained by the competitive pressure from firm 2. For relatively small $k$, competition does not concern firm 1 that much, and the quasi-fixed fee under AUDs extracts most of the buyer's surplus. Thus, the surplus extraction effect dominates the quantity expansion effect, which results lower buyer's surplus. As $k$ increases, because the buyer's outside option becomes better, the surplus extraction effect will be limited and the buyer may not be worse off under AUDs.

□ **Linear Demand Examples.** To illustrate our analyses above and gain further insights on how the limited capacity can affect the equilibrium, we use examples to investigate competitive effects of capacity constraint. We consider a linear demand function $q(p) = 10 - p$, which is generated by the gross benefit function $u(q) = (10 - q/2)q$, with identical zero costs. Assume $k$ is in the interval $(0, 10)$.

Let's take a quick look at some examples of the partial foreclosure effect of AUDs. Table 1 shows the LP and AUD equilibrium outcomes. Using AUDs, firm 1 expands its volume sales dramatically by offering a lower price upon a large threshold. It forces firm 2 to reduce its price by a big percentage in order to stay competitive for the residual demand. As a result, total surplus is increased due to quantity expansion at lower prices. However, firm 2 loses sales, market share, and profit; additionally, the buyer is induced to buy more than under LP and realizes a smaller surplus. For example, at $k = 1$, firm 1's sales with an AUD are more than double what they would be under LP, whereas firm 2's sales are less than $40\%$ of its sales under LP. Firm 1's profit under LP is about half of that under AUDs, whereas firm 2's profit under LP is $30$ times greater than that under AUDs. The buyer's surplus under LP is about $1.6$ times greater than that under AUDs.

[INSERT TABLE 1 HERE]

Now we perform our comparative statics analyses for the full range of $k \in (0, 10)$, by directly applying Propositions 1~2. The computed results are listed in Tables A1 and A2 in Appendix. It is easy to compute the cutoff at which (C2) to be binding is $\widehat{k} \approx 5.354$.

*Firm 2's Volume Sales and Profits.* The equilibrium sales volume for firm 2 under LP and AUD schemes are shown in Figure 3. Firm 2's sales are severely hurt by AUDs. As firm 2 will supply to its full capacity $k$ under LP, the difference between the blue line and red line reveals the idle capacity of firm 2 $k - [q(p_2) - Q]$.

[INSERT FIGURE 3 HERE]

As shown in Figure 4, firm 2's profit is reduced dramatically when firm 1 adopts the AUDs, and this is true for the full range of $k$. So firm 2 gets partially foreclosed by the dominant firm's AUDs for all levels of $k$. This result may raise antitrust concerns when a dominant firm competes against a capacity-constrained competitor and the dominant firm uses the AUDs.

[INSERT FIGURE 4 HERE]

*Buyer's Surpluses.* The equilibrium buyer's surpluses under LP and AUD equilibria are shown in Figure 5. Note that $BS^{AUD}$ crosses $BS^{LP}$ from below at $k \approx 2.3$. So when $k < 2.3$, $BS^{AUD} < BS^{LP}$; when

$k \geq 2.3$, $BS^{AUD} \geq BS^{LP}$. This shows two effects of the AUDs on the buyer. First, the AUD scheme is a more efficient surplus extraction tool than LP, which in principle hurts the buyer. Second, the adoption of AUDs intensifies competition by pushing firm 2 to set a lower price. As shown in Figure 5, when $k$ is relatively small, the former effect dominates the latter because the competitive pressure from firm 2 is limited due to its small capacity; when $k$ is relatively large, the latter effect dominates the former, for more intensified competition becomes significant when firm 2's capacity is large.

[INSERT FIGURE 5 HERE]

From numerical examples above, we find that when $k$ is relatively small, both the competitor and the buyer are hurt by the dominant firm's adoption of the AUDs. This observation appears to be consistent with antitrust concerns put forward in a number of recent cases. Moreover, when $k$ is relatively large, the buyer may not be hurt by the adoption of the AUDs in the short run, but the competitor is still partially foreclosed. So if there are any fixed costs, such limited profit may induce the competitor to exit the market. Hence, the buyer may be further hurt due to the adoption of AUDs by the dominant firm in the long run.

## 6 Conclusion

The use of AUDs by a dominant firm has become a hotly debated topic in antitrust economics and competition policy enforcement. Many antitrust cases involving AUDs share a common feature: a dominant firm's competitors often are small compared with the dominant firm, and thus the dominant firm enjoys some captive portion of a customer's demand. Although the existing literature has thus far focused on interpreting AUDs as a price discrimination tool, investment incentive program, or rent-shifting tool, the antitrust concerns on AUDs are often on its plausible exclusionary effects.

In absence of asymmetric information, downstream competition, or contract externality, we establish strategic effects of AUDs when a dominant firm competes against an equally efficient (or more efficient) but capacity-constrained competitor. We find that the dominant firm is able to use AUDs to partially foreclose its competitor's access to the otherwise competitive portion of the market, when the competitor's capacity is limited. Our findings support the following logic of the European Commission:

> Intel is an unavoidable trading partner. The rebate therefore enables Intel to use the inelastic or "non-contestable" share of the demand of each customer, that is to say the amount that would

> anyhow be purchased by the customer from the dominant undertaking, as leverage to decrease the price of the elastic or "contestable" share of demand, that is to say the amount for which the customer may prefer and be able to find substitutes.
>
> —Intel (Case COMP/C-3/37.990), Commission Decision of 13 May 2009 D(2009) 3726 Final

In a single-product context, we show that the AUDs can serve as a *partial* tying device to leverage dominant firm's market power from its captive portion of the market to the competitive portion. There are a variety of conditional pricing practices. Chao (2013) studies the three-part tariff in a duopoly model. But in his setting, the rival has full capacity to serve the whole market, and competing products are differentiated. Greenlee, Reitman, and Sibley (2008) study bundled loyalty discounts, which requires complete loyalty from consumers when they purchase the tied good in their settings. We consider a single-product model, and our optimal AUDs only require certain amount, not all, of buyer's purchases. Chao, Tan and Wong (2017) examine general nonlinear pricing under complete information. We conjecture that the foreclosure mechanism in this article might work for these other conditional pricing practices. We leave this for future research.

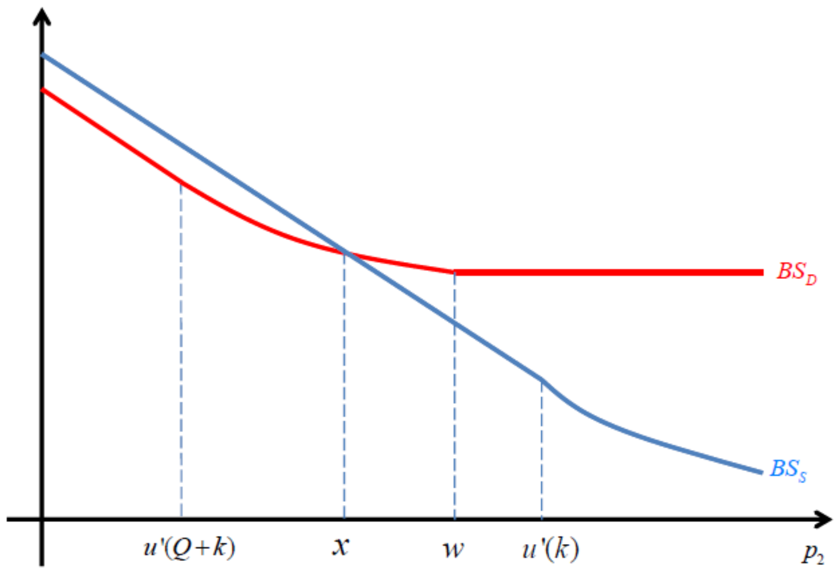


Figure 1: Buyer's Surpluses under AUD

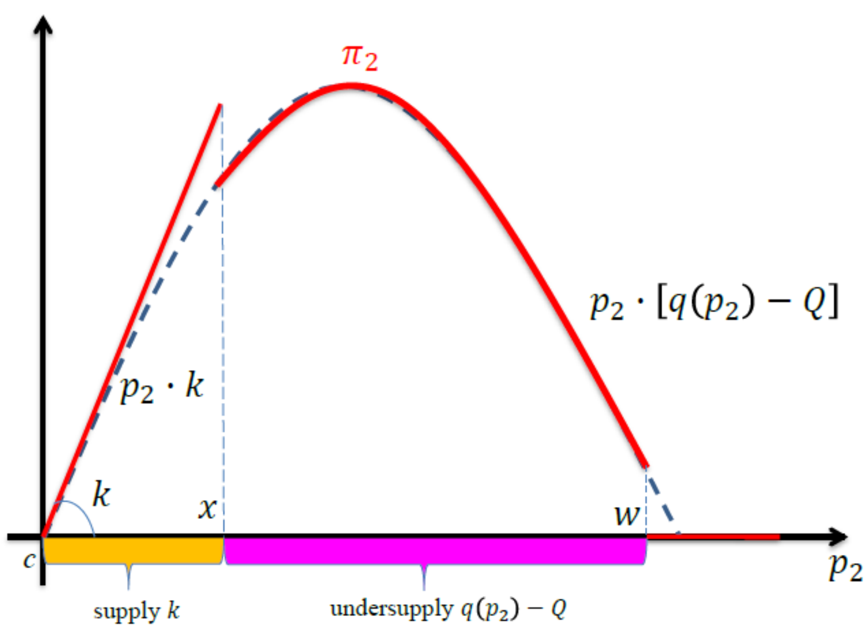


Figure 2: Firm 2's Profit

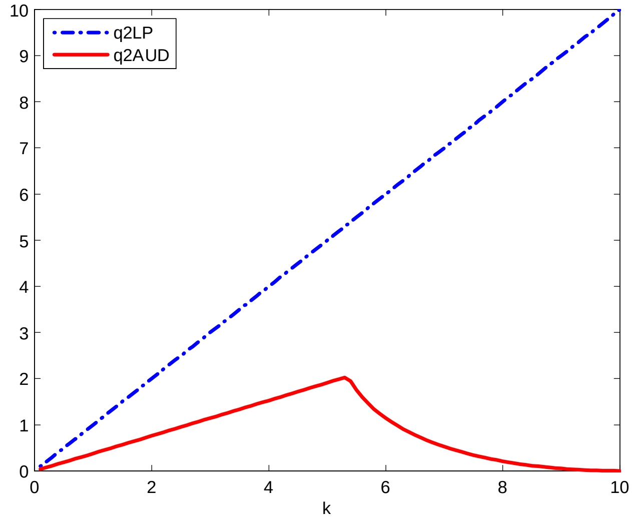


Figure 3: Firm 2's Volume Sales

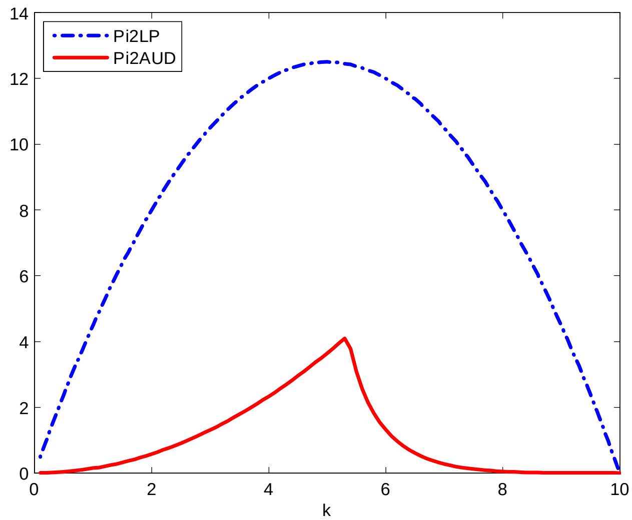


Figure 4: Firm 2's Profits

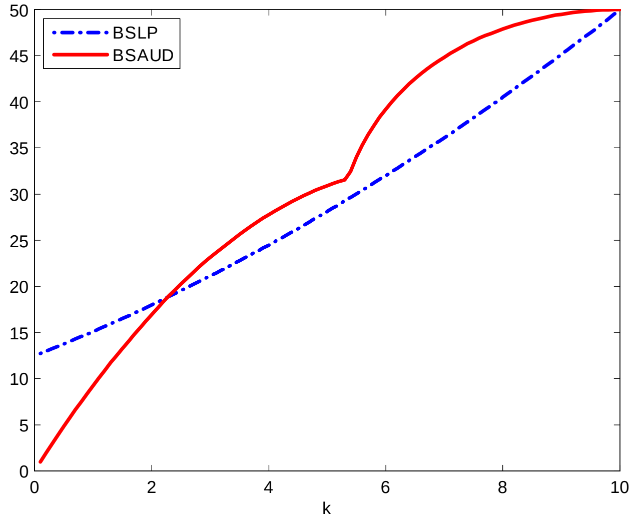


Figure 5: Buyer's Surpluses

**Table 1: Linear Demand Examples**

| | $p_1$ | $p_2$ | $q_1$ | $q_2$ | $\pi_1$ | $\pi_2$ | $BS$ | $TS$ |
|---|---|---|---|---|---|---|---|---|
| $k = 1$ | | | | | | | | |
| **LP** | 4.5 | 4.5 | 4.5 | 1 | 20.25 | **4.5** | **15.13** | 39.87 |
| **AUD** | 4.39 | 0.38 | 9.24 | 0.38 | 40.55 | **0.15** | **9.24** | 49.93 |
| $\Delta\%$ | $-2$ | $-92$ | $+105$ | $-62$ | $+100$ | $-97$ | $-39$ | $+25$ |
| $k = 2$ | | | | | | | | |
| **LP** | 4 | 4 | 4 | 2 | 16 | **8** | **18** | 42 |
| **AUD** | 3.80 | 0.76 | 8.47 | 0.77 | 32.18 | **0.58** | **16.94** | 49.71 |
| $\Delta\%$ | $-5$ | $-81$ | $+112$ | $-62$ | $+101$ | $-93$ | $-6$ | $+18$ |

## Appendix

**Proof of Proposition 1.** First, firm 2 must set $p_2 \leq p_1$ unless $p_1 < 0$, because otherwise firm 2 would have no sales and thus zero profit. But $p_1 < 0$ can be ruled out as it gives firm 1 negative profit. So $p_2 \leq p_1$.

Second, firm 1 must set $p_1 < u'(k)$. This is because, due to $p_2 \leq p_1$, the buyer always buys from firm 2 first, and $u'(k) \leq p_1$ would result in no sale for firm 1.

Hence, with $p_2 \leq p_1 < u'(k)$, the buyer buys $k$ from firm 2 at $p_2$ and $q(p_1) - k$ from firm 1 at $p_1$. Firm 2's profit is $p_2 \cdot k$ and firm 1's profit is $p_1 \cdot [q(p_1) - k]$. It is easy to see that firm 2 must set $p_2 = p_1$ and firm 1 will set $p_1 = p^R(k)$. ■

**Proof of Lemma 1.** (i) Under AUDs, if $q_1 < Q$, then $q_1 = 0$ and $\pi_1 = 0$ because $p_o = \infty$; if $q_1 > Q$, then it is equivalent to LP ($p_1$) vs. LP ($p_2$), and the AUDs cannot improve firm 1's profit.

We now show $Q < q^e$. Suppose not, i.e., $u'(Q) \leq 0$. Then under DS, firm 2 would have no sales, and it would try its best to undercut until $0$ in order to induce SS, if possible. To ensure the buyer meets $Q$, firm 1 must make $u(Q) - p_1 \cdot Q \geq u(k)$, i.e., $p_1 \cdot Q \leq u(Q) - u(k)$. Thus,

$$\begin{aligned}\pi_1 &= p_1 \cdot Q \\ &\leq u(Q) - u(k) \\ &\leq V(0) - u(k) = S^{cap}.\end{aligned}$$

So in order to have a strictly profitable improvement over $S^{cap}$, we must have $Q < q^e$.

(ii) $Q < q(p_2)$ follows from the fact that $0 < u'(Q)$ and $u'(Q) \leq p_2$ would result in no sales for firm 2.

We now show $q(p_2) < Q + k$. Suppose not, i.e., $p_2 \leq u'(Q + k)$. It follows that $\pi_2 = p_2 \cdot k \leq u'(Q+k) \cdot k$. Then firm 2 can always increase its profit without losing any sales, as long as $p_2 < u'(Q+k)$. Next, we rule out the case of $p_2 = u'(Q + k)$. Suppose $BS_D(u'(Q + k)) \geq BS_S(u'(Q + k))$, i.e., $u(Q + k) - p_1 \cdot Q - u'(Q + k) \cdot k \geq u(k) - u'(Q + k) \cdot k$. So $p_1 \cdot Q \leq u(Q + k) - u(k)$. Then

$$\begin{aligned}\pi_1 &= p_1 \cdot Q \\ &\leq u(Q + k) - u(k) \\ &\leq V(0) - u(k) = S^{cap}.\end{aligned}$$

For $\pi_1 > S^{cap}$, we must have $BS_D(u'(Q+k)) < BS_S(u'(Q+k))$, but then the buyer would choose SS. Thus, in order to induce the buyer to choose DS, firm 1 has to ensure $u'(Q+k) < p_2$. ■

**Proof of Lemma 2.** It is easy to see $p_2 \leq p_1$ under AUDs, because otherwise firm 2 would have no sales under DS. In the following, we only show $p_1 < u'(k)$, based on the idea that if $u'(k) \leq p_1$, then $BS_S(w) \geq BS_D(w)$, which implies $BS_S(p_2) \geq BS_D(p_2)$ for $p_2 \leq w$, because from (4) and (6), $\partial BS_S/\partial p_2 \leq \partial BS_D/\partial p_2 \leq 0$. That is, if $u'(k) \leq p_1$, then the buyer always chooses SS when $p_2 \leq w$.

Suppose $u'(k) \leq p_1$. When $q(w) \leq k$, $BS_S(w) = V(w)$, $BS_D(w) = V(w) + (w - p_1) \cdot Q$. Because $w \leq p_1$, we have $BS_S(w) \geq BS_D(w)$. Recall that $w = \min\{p_1, u'(Q)\}$. $k < q(w)$ must imply $w = u'(Q)$ because our supposition $u'(k) \leq p_1$. Therefore, when $k < q(w)$, we must have $k < Q$. It follows that

$$\begin{aligned} BS_S(w) &= u(k) - u'(Q) \cdot k \\ &> u(Q) + u'(k) \cdot (k - Q) - u'(Q) \cdot k \\ &= u(Q) + [u'(k) - u'(Q)] \cdot k - u'(k) \cdot Q \\ &> u(Q) - p_1 Q \\ &= BS_D(w), \end{aligned}$$

where the first inequality follows from $u''(\cdot) < 0$ and the second inequality follows from $k < Q$, $u'(k) \leq p_1$ and $u''(\cdot) < 0$. ■

**Proof of Lemma 3.** First, we show that $BS_D(w) > BS_S(w)$. Suppose not. Then $BS_D(w) \leq BS_S(w)$ implies $BS_D(p_2) \leq BS_S(p_2), \forall p_2 \leq w$, because $\partial BS_S/\partial p_2 \leq \partial BS_D/\partial p_2 \leq 0$. We have $p_2 \leq w$ in equilibrium because otherwise firm 2 would have no sales. It follows that the buyer would always choose SS from firm 2 when $BS_D(w) \leq BS_S(w)$. Thus, in order to induce DS, we must have $BS_D(w) > BS_S(w)$.

Recall from the proof of Lemma 1 that, for $\pi_1 > S^{cap}$, we must have $BS_D(u'(Q+k)) < BS_S(u'(Q+k))$. Combining it and $BS_D(w) > BS_S(w)$ with $\partial BS_S/\partial p_2 \leq \partial BS_D/\partial p_2 \leq 0$, the unique intersection follows.

In the AUD equilibrium, we must have $x < p_2$, because otherwise firm 1 would have no sales. $x < p_2$, (C2) and (C1) together yield $x < p_2 \leq p_1 < u'(k)$. Because now $x < u'(k)$, $\overline{q}(k, x) = k$ all the time. So the determination (9) follows. ■

**Proof of Lemma 4.** First, in the AUD equilibrium, $x \leq p_2 \leq w$. The first inequality holds because otherwise the buyer would SS and firm 1 would have no sale. The second inequality follows because otherwise firm 2 would have no sales.

To ensure firm 2 chooses $p_2$ s.t. $x \leq p_2 \leq w$, we must have (11). Note that firm 2's profit has a drop at $p_2 = x$, i.e., $x \cdot k > x \cdot [q(x) - Q]$, because $u'(Q+k) < x$. In order to have (11), we must have the optimal $p_2$ to $\max_{x \leq p_2 \leq w} p_2 \cdot [q(p_2) - Q]$ as an interior solution, i.e., $x < p_2 \leq w$. The first-order condition for an interior solution satisfies (13). Clearly, $\max_{p_2 < x} p_2 \cdot k = x \cdot k$.

Next, we show that (12) holds in equilibrium. Using (10),

$$\pi_1 = p_1 \cdot Q = x \cdot Q + V(x) - [u(k) - x \cdot k],$$

$$\frac{\partial \pi_1}{\partial x} = Q + k - q(x) > 0,$$

where the inequality follows from $u'(Q+k) < x$ and $Q > 0$. Consequently, as long as (11) is not binding, $\pi_1$ can always be increased by increasing $x$. Thus, (11) must be binding, and thereby (12) follows. ■

To prove Proposition 2, we first establish two lemmas. Lemma A.1 shows that, ignoring (C1) and (C2), $\pi_1^{AUD}(Q)$ is single-peaked in $Q$, and the peak satisfies all other constraints.

**Lemma A.1** *For any $k \in (0, q^e)$, there exists a unique $Q(k)$ that satisfies (FOC), (9), (12), (13), and (14). Moreover, $Q(k)$ is strictly decreasing in $k$.*

Thus, when neither (C1) nor (C2) is binding, such a peak maximizes $\pi_1^{AUD}(Q)$. So the key question is when and which of the constraints (C1) and (C2) will be binding. Lemma A.2 offers an answer to it.

**Lemma A.2 (When (C2) is binding)** *Given $(Q, p_1, p_2, x)$ jointly determined by (9), (12), (13), and (FOC),*

*(i) $u'(Q+k) < x < u'(k)$ implies (C1);*

*(ii) there exists a unique $\widehat{k} \in (0, q^e)$ such that (C2) is binding if and only if $k \geq \widehat{k}$.*

Part (i) of the lemma says that, given other constraints hold, (C1) is redundant. So the only possible binding constraint is (C2). Part (ii) tells us that, (C2) is binding only for $k$ above $\widehat{k}$.

**Proof of Lemma A.1.** Here we first show the existence, uniqueness and sufficiency of (FOC), and then we prove the solution to it satisfies the inequality constraint (14). Last, we show $Q'(k) < 0$.

*Step 1: Existence, Uniqueness and Sufficiency of (FOC)*

From (12), $x(Q) = \frac{\pi^R(Q)}{k}$ for all $Q \in [0, q^e]$. Note that

$$\frac{d\pi_1^{AUD}}{dQ} = \frac{p_2}{k} \cdot \varphi(Q), \tag{16}$$

where $\varphi(Q) \equiv q(x) + q(p_2) - 2Q - k$.

Let $Q_k \equiv \pi'(p_k)$ with $p_k$ satisfying $k + p_k \cdot q'(p_k) = 0$.

Next we show that $\varphi'(Q) < 0$ for $Q \in [Q_k, q^e]$ and there exists a unique $Q(k) \in [Q_k, q^e]$ s.t. $\varphi(Q(k)) = 0$. These together imply that $\pi_1^{AUD}$ is single-peaked in $Q$, and $Q(k)$ is such a unique peak.

$$\begin{aligned}\varphi'(Q) &= q'(x) \cdot x'(Q) + q'(p_2) \cdot p_2'(Q) - 2 \\ &= q'(x) \cdot (-\frac{p_2}{k}) + q'(p_2) \cdot \frac{1}{\pi''(p_2)} - 2 \\ &= [\frac{q'(p_2)}{\pi''(p_2)} - 1] - \frac{k + p_2 \cdot q'(x)}{k},\end{aligned}$$

where the second equality follows from (12) and (13).

From $q''(\cdot) \leq 0$, $\pi''(p_2) = 2q'(p_2) + p_2 \cdot q''(p_2) < q'(p_2) < 0$. Hence, $0 < \frac{q'(p_2)}{\pi''(p_2)} < 1$. For $Q > Q_k$, we have $p_2 = p_2(Q) < p_2(Q_k) = p_k$, thereby $k + p_2 \cdot q'(p_2) > 0$. Note that $x < p_2$ for $Q > Q_k$, which follows from (13). Hence, $k + p_2 \cdot q'(x) \geq k + p_2 \cdot q'(p_2) > 0$ for $Q > Q_k$ follows from $x < p_2$ and $q'' \leq 0$. As a result, we have $\varphi'(Q) < 0$ for $Q \in [Q_k, q^e]$.

Last, we show that $\varphi(Q)$ does cross zero from above. At $Q = q^e$, $p_2(q^e) = x(q^e) = 0$, thus $\varphi(q^e) = -k < 0$. At $Q = Q_k$, $x = p_2 = p_k$, thus $\varphi(Q_k) = 2 \cdot [q(p_k) - Q_k] - k = k \geq 0$ with "=" only if $k = 0$. So there exists a unique $Q(k) \in [Q_k, q^e)$ s.t. $\varphi(Q(k)) = 0$.

*Step 2: Check Constraints* $u'(Q + k) < x < p_2 < u'(Q)$

Note that $x < p_2$ has been shown in Step 1 for $Q(k) > Q_k$, and that $p_2 < u'(Q(k))$ follows from (13). Moreover, $u'(Q(k) + k) < x$ follows from (FOC), because $Q(k) + k - q(x) = q(p_2) - Q(k) > 0$ due to $p_2 < u'(Q(k))$.

*Step 3:* $Q'(k) < 0$

Total differentiate (FOC) w.r.t. $k$, we have

$$\begin{aligned}
\varphi'(Q) \cdot Q'(k) + \frac{\partial \varphi}{\partial x} \cdot \frac{\partial x}{\partial k} + \frac{\partial \varphi}{\partial k} = 0 \\
\therefore \varphi'(Q) \cdot Q'(k) = 1 - q'(x) \cdot \frac{\partial x}{\partial k} \\
= \frac{k + x \cdot q'(x)}{k},
\end{aligned}$$

where the last equality follows from (12). Recall that in Step 1, we have shown that $\varphi'(Q) < 0$ for $Q \in [Q_k, q^e]$. Moreover, $k + x \cdot q'(x) > k + p_2 \cdot q'(p_2) > 0$ for all $Q \in [Q_k, q^e]$, where the first inequality follows from $q'' \leq 0$ and $x < p_2$, and the second one follows from $Q(k) \geq Q_k$ and the definition of $Q_k$ in Step 1. Thus, $Q'(k) < 0$. ■

**Proof of Lemma A.2.** (i) From (10) $p_1 \cdot Q = x \cdot Q + V(x) - [u(k) - x \cdot k]$, we have

$$\begin{aligned}
[p_1 - u'(k)] \cdot Q &= [x - u'(k)] \cdot Q + V(x) - [u(k) - x \cdot k] \\
&< [x - u'(k)] \cdot Q + [u'(k) - x] \cdot [q(x) - k] \\
&= [u'(k) - x] \cdot [q(x) - k - Q] \\
&< 0,
\end{aligned}$$

where the concavity of $u(q)$ leads to the first inequality, and the second inequality follows from $u'(Q+k) < x < u'(k)$.

(ii) Let

$$\begin{aligned}
D(k) &\equiv \pi_1^{AUD}(Q) - p_2 \cdot Q \\
&= V(x) - [u(k) - x \cdot k] - (p_2 - x) \cdot Q,
\end{aligned}$$

where $(Q(k), p_2(k), x(k))$ is jointly determined by (12), (13), and (FOC). Then, (C2) becomes $D(k) \geq 0$.

Here we first show that $D(0) > 0$ and $D(k) < 0$ for $\alpha \leq k$, then we prove $D(k)$ decreases with $k$ for $k \leq \alpha$ ($\alpha$ shall be defined in Step 1 below). Hence, we conclude with the existence of $\widehat{k} \in (0, \alpha)$ s.t. $D(k) \gtreqless 0$ for $k \lesseqgtr \widehat{k}$.

*Step 1:* $D(0) > 0$ *and* $D(k) < 0$ *for* $\alpha \leq k$.

When $k = 0$, $x = p_2 = 0$. $D(0) = V(0) > 0$.

From (13), $p_2$ decreases with $Q$. Combining with $Q'(k) < 0$ from Lemma A.1, we have $p_2(k)$ increases with $k$. From the concavity of $u(q)$, $u'(k)$ decreases with $k$. $p_2(0) = 0 < u'(0)$, $p_2(q^e) > 0 = u'(q^e)$. Thus, there exists a unique $\alpha \in (0, q^e)$ s.t. $p_2(k) \lesseqqgtr u'(k)$ for $k \lesseqqgtr \alpha$.

For $k \geq \alpha$, when $x \leq u'(k)$,

$$\begin{aligned}
D(k) &= V(x) - [u(k) - x \cdot k] - (p_2 - x) \cdot Q \\
&< [u'(k) - x] \cdot [q(x) - k] - (p_2 - x) \cdot \pi'(p_2) \text{ } (\because u''(q) < 0 \text{ and } x \leq u'(k)) \\
&= [u'(k) - x] \cdot [\pi'(p_2) + p_2 \cdot q'(p_2)] - (p_2 - x) \cdot \pi'(p_2) \text{ (by (FOC))} \\
&= [u'(k) - p_2] \cdot \pi'(p_2) + [u'(k) - x] \cdot p_2 \cdot q'(p_2) \text{ } (\because x \leq u'(k) \leq p_2) \\
&< 0
\end{aligned}$$

For $k \geq \alpha$, when $x > u'(k)$,

$$\begin{aligned}
D(k) &< [u'(k) - p_2] \cdot \pi'(p_2) + [u'(k) - x] \cdot p_2 \cdot q'(p_2) \\
&= -\{[p_2 - u'(k)] \cdot \pi'(p_2) - [x - u'(k)] \cdot [-p_2 \cdot q'(p_2)]\}.
\end{aligned}$$

If we can show that $[p_2 - u'(k)]/[x - u'(k)] > [-p_2 \cdot q'(p_2)]/\pi'(p_2)$, then $D(k) < 0$.

Because $u'(k) > 0$ and $p_2 > x$, $[p_2 - u'(k)]/[x - u'(k)] > p_2/x$. Thus, to show $D(k) > 0$, it suffices to show

$$\frac{p_2}{x} \geq \frac{-p_2 \cdot q'(p_2)}{\pi'(p_2)},$$

which is reduced to

$$\pi'(p_2) + x \cdot q'(p_2) \geq 0.$$

$$\begin{aligned}
\pi'(p_2) + x \cdot q'(p_2) &= q(x) - k - (p_2 - x) \cdot q'(p_2) \text{ (by (FOC))} \\
&\geq (p_2 - x) \cdot [-q'(p_2)] - [x - u'(k)] \cdot [-q'(x)] \text{ } (\because q''(p) \leq 0)
\end{aligned}$$

Because $q''(p) \leq 0$ and $p_2 > x$, $-q'(p_2) \geq -q'(x)$. If we can show that $p_2 - x > x - u'(k)$, then $\pi'(p_2) + x \cdot q'(p_2) \geq 0$.

Because $u'(k) > 0$,

$$p_2 - x > x \tag{17}$$

would imply $p_2 - x > x - u'(k)$.

(12) and (FOC) together imply (17) as follows. (12) gives

$$\begin{aligned}\frac{p_2}{x} &= \frac{k}{-p_2 \cdot q'(p_2)} \\ &= \frac{q(x) - q(p_2) - 2 \cdot p_2 \cdot q'(p_2)}{-p_2 \cdot q'(p_2)} \text{ (by (FOC))} \\ &= 2 + \frac{q(x) - q(p_2)}{-p_2 \cdot q'(p_2)} \\ &> 2 \text{ } (\because x < p_2)\end{aligned}$$

Hence, $D(k) < 0$ for $k \geq \alpha$, when $x > u'(k)$. This completes Step 1.

*Step 2: $D'(k) < 0$ for $k \leq \alpha$.*

Using (FOC),

$$\begin{aligned}D'(k) &= \frac{\partial D}{\partial k} + \frac{\partial D}{\partial x} \cdot \frac{\partial x}{\partial k} + \frac{\partial D}{\partial p_2} \cdot p_2'(k) \\ &= x - u'(k) + [k + \pi'(p_2) - q(x)] \cdot (-\frac{x}{k}) - [\pi'(p_2) + p_2\pi''(p_2)] \cdot p_2'(k) \\ &= x - u'(k) + [k - q(x)] \cdot p_2'(k) + p_2 \cdot q'(p_2) \cdot \frac{x}{k} \\ &+ p_2 \cdot [-q'(p_2) - p_2 q''(p_2)] \cdot p_2'(k) \text{ (by (FOC))} \\ &\leq [k - q(x)] \cdot p_2'(k) + \underbrace{(x - p_2)}_{\langle 1 \rangle} + \underbrace{p_2 \cdot q'(p_2) \cdot \frac{x}{k}}_{\langle 2 \rangle} \\ &+ \underbrace{p_2 \cdot [-q'(p_2) - p_2 q''(p_2)] \cdot p_2'(k)}_{\langle 3 \rangle},\end{aligned}$$

where the inequality follows from $p_2 \leq u'(k)$ for $k \leq \alpha$. Note that $[k - q(x)] \cdot p_2'(k)$ is negative, because $x < u'(k)$ for $k \leq \alpha$ and $p_2'(k) > 0$. So if we can show $\langle 1 \rangle + \langle 2 \rangle + \langle 3 \rangle$ is negative, then this part is complete.

From (FOC),

$$p_2'(k) = \frac{k + x\cdot q'(x)}{-\pi''(p_2)\cdot[k + p_2\cdot q'(x)] + k\cdot[-q'(p_2) - p_2\cdot q''(p_2)]}$$
$$< \frac{k + x\cdot q'(x)}{2k + p_2\cdot q'(x)} \cdot \frac{1}{-q'(p_2) - p_2\cdot q''(p_2)}, \tag{18}$$

where the inequality follows from $-\pi''(p) > -q'(p) - p\cdot q''(p)$.

Hence,

$$\begin{aligned}
\langle 2\rangle + \langle 3\rangle &< p_2\cdot q'(p_2)\cdot\frac{x}{k} + p_2\cdot\frac{k + x\cdot q'(x)}{2k + p_2\cdot q'(x)} \\
&= \frac{p_2\cdot q'(p_2)\cdot x\cdot 2k - x\cdot q'(x)\cdot x\cdot k + k\cdot p_2\cdot[k + x\cdot q'(x)]}{k\cdot[2k + p_2 q'(x)]} \\
&= \frac{p_2\cdot[k + 2\cdot x\cdot q'(p_2)] + x\cdot q'(x)\cdot(p_2 - x)}{2k + p_2\cdot q'(x)},
\end{aligned}$$

where the first inequality follows from (18), and the last equality is from (9). Therefore,

$$\begin{aligned}
&\langle 1\rangle + \langle 2\rangle + \langle 3\rangle \\
&< (x - p_2) + \frac{p_2\cdot[k + 2x\cdot q'(p_2)] + x\cdot q'(x)\cdot(p_2 - x)}{2k + p_2\cdot q'(x)} \\
&= \frac{(p_2 - 2x)[q(p_2) - q(x)] + 2(p_2 - x)\cdot p_2\cdot q'(p_2) - (p_2 - x)^2\cdot q'(x)}{2k + p_2\cdot q'(x)} \\
&\leq \frac{(p_2 - 2x)q'(x)(p_2 - x) + 2(p_2 - x)\cdot p_2\cdot q'(p_2) - (p_2 - x)^2\cdot q'(x)}{2k + p_2\cdot q'(x)} \\
&= \frac{p_2 - x}{2k + p_2\cdot q'(x)}\cdot[p_2\cdot q'(p_2) + p_2\cdot q'(p_2) - x\cdot q'(x)],
\end{aligned}$$

where the first equality follows from (FOC), and the second inequality is due to $q''(p) \leq 0$ and $x < p_2$. Indeed, $2k + p_2\cdot q'(x) > k + p_2\cdot q'(x) > k + p_2\cdot q'(p_2) > 0$, where the second inequality follows from $q''(p) \leq 0$ and $x < p_2$. Because $p\cdot q'(p)$ is decreasing in $p, \forall p > 0$, $p_2\cdot q'(p_2) - x\cdot q'(x) < 0$ as $x < p_2$. Thus, $p_2\cdot q'(p_2) + p_2\cdot q'(p_2) - x\cdot q'(x) < 0$, thereby Step 2 is completed.

*Step 3: There exists a unique* $\widehat{k} \in (0, \alpha)$ *s.t.* $D(k) \gtreqless 0$ *for* $k \lesseqgtr \widehat{k}$.

This follows directly from Steps 1 and 2. ■

**Proof of Proposition 2.** With Lemmas 2 and A.2, we know that when $k < \widehat{k}$, the equilibrium outcome $(Q, p_1, p_2)$ along with threat price $x$ is jointly determined by (9), (12), (13), and (FOC), with (FOC) being replaced by (C2) when $\widehat{k} \leq k$. The sufficiency of (FOC) has already been shown in the proof of Lemma A.1. In the proof of Lemma A.2, $u'(Q+k) < x < u'(k)$ ensures that $p_1 < u'(k)$. And we know that $u'(Q+k) < x < u'(k)$ is true under (FOC) for $k < \widehat{k}$. So here we only need to show the existence of equilibrium when $\widehat{k} \leq k$, and check the constraint $u'(Q+k) < x < u'(k)$.

*Step 1: Existence of the Solution to $p_2 = p_1$ when $\widehat{k} \leq k$.*

Similar to $D(k)$ but without using the equilibrium $Q(k)$ from (FOC), we can define

$$\begin{aligned} d(Q,k) &\equiv \pi_1^{AUD}(Q) - p_2 \cdot Q \\ &= V(x) - [u(k) - x \cdot k] - (p_2 - x) \cdot Q, \end{aligned}$$

where $(p_2(Q), x(Q))$ is jointly determined by (12) and (13). So $D(k) = d(Q(k), k)$, and the constraint (C2) $p_2 \leq p_1$ is $d(Q,k) \geq 0$.

When $\widehat{k} \leq k$, $d(Q(k), k) = D(k) < 0$. At $Q = q^e$, (12) and (13) lead to $x = p_2 = 0$. So $d(q^e, k) = V(0) - u(k) > 0$. From the continuity of $d(Q,k)$, there must exist a $\overline{Q}(k) \in (Q(k), q^e)$ s.t. $d(\overline{Q}(k), k) = 0$, i.e., $p_2(\overline{Q}(k)) = p_1(\overline{Q}(k))$.

*Step 2: Check Constraints $u'(Q+k) < x < p_2 < u'(Q)$ and $p_1 < u'(k)$*

Because $d(\overline{Q}(k), k) = 0$, we have $(p_2 - x) \cdot \overline{Q} = V(x) - [u(k) - x \cdot k] > 0$. So $p_2 > x$.

In Step 1 of the proof of Lemma A.1, when showing the sufficiency of (FOC), we proved $\varphi'(Q) < 0$ for any $Q > Q_k$. Because $Q_k < Q(k) < \overline{Q}(k)$, we have $\varphi(\overline{Q}) = q(x) + q(p_2) - 2\overline{Q} - k < 0$, which is equivalent to $q(p_2) - \overline{Q} < k + \overline{Q} - q(x)$. Hence, for $u'(\overline{Q} + k) < x$ and $p_2 < u'(\overline{Q})$, it suffices to show that $q(p_2) < \overline{Q}$, which follows from (13).

From the proof of Lemma A.2, $u'(\overline{Q} + k) < x < u'(k)$ ensures that $p_1 < u'(k)$. ■

**Proof of Corollary 2.** When $k < \widehat{k}$,

$$\begin{aligned} Q + k &= q(x) + q(p_2) - Q \text{ (By (FOC))} \\ &= q(x) - p_2 \cdot q'(p_2) \text{ (By (13))} \\ &\geq q(x) - x \cdot q'(x) \\ &\geq q^e, \end{aligned}$$

where the first inequality is from $p_2 \cdot q'(p_2)$ decreases with $p_2$ for $0 \leq p_2$ and $x \leq p_2$, and the second inequality follows from $q(x) - x \cdot q'(x)$ is weakly increasing in $x$ for $0 \le x$. Note that "=" occurs only when $x = p_2 = 0$, that is, only when $k = 0$.

When $k \ge \widehat{k}$, (FOC) is replaced by (C2). So the equilibrium solution $\overline{Q}(k) > Q(k)$, where $Q(k)$ is characterized by (FOC).

$$\begin{aligned}\overline{Q} + k &> q(x) + q(p_2) - \overline{Q} \text{ (By } \varphi'(Q) < 0 \text{ and } \overline{Q}(k) > Q(k)) \\ &= q(x) - p_2 \cdot q'(p_2) \text{ (By (13))} \\ &\ge q(x) - x \cdot q'(x) \\ &\ge q^e,\end{aligned}$$

where the first inequality is from $p_2 \cdot q'(p_2)$ decreases with $p_2$ for $0 \le p_2$ and $x \le p_2$, and the second inequality follows from $q(x) - x \cdot q'(x)$ is weakly increasing in $x$ for $0 \le x$. ■

**Proof of Corollary 3.** (i) From Lemma A.1, we have $Q'(k) < 0$ for $k \le \widehat{k}$.

Total output is $q(p_2)$. $p_2(Q)$ decreases with $Q$ from (13). Because $Q(k)$ decreases with $k$, $p_2(k)$ must increase with $k$ for $k \le \widehat{k}$. So total output $q(p_2)$ decreases with $k$ for $k \le \widehat{k}$.

(ii) The comparative statics on $p_2(k)$ is shown in Part (i). Similarly, we can derive the same result for $x(k)$ from (12).

(iii) For $k \le \widehat{k}$, $\pi_1^{AUD} = V(x) - [u(k) - xk] + x \cdot Q$, and we have $u'(q + k) < x < u'(k)$. Thus

$$\begin{aligned}\frac{d\pi_1^{AUD}}{dk} &= [k + Q - q(x)] \cdot \frac{\partial x}{\partial k} - [u'(k) - x] \\ &= [k + Q - q(x)] \cdot (-\frac{x}{k}) - [u'(k) - x] \text{ (By (12))} \\ &< 0.\end{aligned}$$

Because $\pi_2^{AUD} = \pi^R(Q)$ decreases with $Q$, this part follows from Part (i).

(iv) $TS^{AUD} = u(q(p_2))$. $\frac{dTS^{AUD}}{dp_2} = u'(q(p_2)) \cdot q'(p_2) < 0$. Then this part follows from the result on $p_2(k)$ of Part (ii). ■

**Proof of Proposition 3.** (i) Note that $p_2(0) = 0 < p^m = p^R(0)$, and $p_2(\alpha) = u'(\alpha) > p^R(\alpha)$ ($\because \pi'(p) < q(p)$). Moreover, $p_2'(k) > 0$ and $p^{R\prime}(k) < 0$. Hence, $\exists$ a unique $k_0 \in (0, \alpha)$ s.t. $p_2(k) \lesseqqgtr p^R(k), \forall k \lesseqqgtr k_0$.

(ii) $q_1^{AUD} = Q > q^e - k > q(p^R) - k = q_1^{LP}$ follows from Corollary 2. $q_2^{AUD} = q(p_2) - Q < k = q_2^{LP}$

follows from the fact that $u'(Q+k) < p_2$.

(iii) It is obvious that $\pi_1^{AUD} > \pi_1^{LP}$. $\pi_2^{AUD} = p_2 \cdot [q(p_2) - Q] < p_2 \cdot k < p_2^{LP} \cdot k = \pi_2^{LP}$, where the first inequality follows from the fact that $u'(Q+k) < p_2$ and the second one follows from $p_2^{AUD} < p_2^{LP}, \forall k < \min\{k_0, \widehat{k}\}$.

(iv) When $k \to 0$, $BS^{AUD} \to 0$, and $BS^{LP} \to V(p^m) > 0$ as $p^R \to p^m$. From continuity of $BS^{AUD}$ and $BS^{LP}$, this part follows.

(v) $TS^{LP} = u(q(p^R))$. Because $p^{R\prime}(k) < 0$ and $q(p^R) < 0$, $\frac{dTS^{LP}}{dk} = u'(q(p^R)) \cdot q'(p^R) \cdot p^{R\prime}(k) > 0$. $TS^{AUD} = u(q(p_2))$. Because $p_2'(k) > 0$ for $k < \widehat{k}$, $\frac{dTS^{AUD}}{dk} = u'(q(p_2)) \cdot q'(p_2) \cdot p_2'(k) < 0$ for $k < \widehat{k}$.

At $k = 0$, $p^R = p^m > 0 = p_2$. So $TS^{AUD} = V(0) > u(q^m) = TS^{LP}$. At $k = q^e$, $p^R = 0 = p_2$. So $TS^{AUD} = V(0) = TS^{LP}$. Consequently, there exists a $k_2 \in (0, q^e]$ s.t. $TS^{AUD} > TS^{LP}$ for $k < k_2$. ■

**Table A1: Equillibrium Tariffs for Linear Demand**

| | **LP** | **AUD** |
|---|---|---|
| **Quantity Threshold** | N/A | $10-(3-\sqrt{5})k$ when $k < \widehat{k}$<br>$10-2a$ when $k \geq \widehat{k}$ |
| **Firm 1's Per-Unit Price** | $\frac{10-k}{2}$ | $\frac{50-10\cdot k+\frac{5\sqrt{5}-9}{4}\cdot k^2}{10-(3-\sqrt{5})\cdot k}$ when $k < \widehat{k}$<br>$a$ when $k \geq \widehat{k}$ |
| **Firm 2's Per-Unit Price** | $\frac{10-k}{2}$ | $\frac{3-\sqrt{5}}{2} \cdot k$ when $k < \widehat{k}$<br>$a$ when $k \geq \widehat{k}$ |

**Table A2: Equillibrium Surpluses for Linear Demand**

| | **LP** | **AUD** |
|---|---|---|
| **Firm 1's Profit** | $\frac{(10-k)^2}{4}$ | $50 - 10 \cdot k + \frac{5\sqrt{5}-9}{4} \cdot k^2$ when $k < \widehat{k}$<br>$a(10-2a)$ when $k \geq \overline{k}$ |
| **Firm 2's Profit** | $\frac{k(10-k)}{2}$ | $\frac{7-3\sqrt{5}}{2} \cdot k^2$ when $k < \widehat{k}$<br>$a^2$ when $k \geq \widehat{k}$ |
| **Buyer's Surplus** | $\frac{(10+k)^2}{8}$ | $k \cdot [10-(3-\sqrt{5})k]$ when $k < \widehat{k}$<br>$\frac{(10-a)^2}{2}$ when $k \geq \widehat{k}$ |
| **Total Surplus** | $\frac{(10+k)(30-k)}{8}$ | $50 - \frac{7-3\sqrt{5}}{4} \cdot k^2$ when $k < \widehat{k}$<br>$\frac{100-a^2}{2}$ when $k \geq \widehat{k}$ |

Note: In Tables A1 and A2, $\widehat{k} = \frac{5}{29} \cdot (30 + 8\sqrt{5} - \sqrt{118 + 74\sqrt{5}}) \simeq 5.3538$.
$a$ is determined by $a(a^3 - 4k \cdot a^2 + 6k^2 \cdot a - 20k^2) + k^2(10-k)^2 = 0$ ($a < \min\{k, 10-k\}$).

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