

Asymmetry in Capacity and the Adoption of All-Units Discounts*

Yong Chao Guofu Tan Adam Chi Leung Wong[†]

March 22, 2019

Abstract

In many abuse of dominance antitrust cases, the dominant firm adopts pricing schemes involving all-units discounts, whereas its smaller competitors often use simple linear pricing. We provide a game-theoretic justification for the observed asymmetry in pricing practices by studying a model in which a firm with full capacity faces a capacity-constrained rival. The asymmetry in capacity between the firms, which gives rise to the captive market, allows the dominant firm to take advantage of the quantity commitment through all-units discounts while the capacity-constrained rival is induced to offer simple linear pricing.

Keywords: linear pricing, all-units discounts, capacity constraint

JEL codes: D43, L13, L42

*We thank editor Yongmin Chen and two anonymous referees for valuable suggestions that have significantly improved this paper. We also gratefully acknowledge helpful comments from Zhiqi Chen, Benjamin Hermalin, Patrick Legros, Ping Lin, and seminar participants at Penn State University, 2018 IWIE Conference at Nanjing University, and 2018 China Meeting of Econometric Society at Fudan University. This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

[†]**Chao:** College of Business, University of Louisville. Address: 2301 S. 3rd Street, Louisville, KY 40292, United States. Email: yong.chao@louisville.edu. **Tan:** Department of Economics, University of Southern California. Address: 3620 S. Vermont Avenue, Los Angeles, CA 90089, United States. Email: guofutan@usc.edu. **Wong** (Corresponding author): Department of Economics, Lingnan University. Address: 8 Castle Peak Road, Tuen Mun, Hong Kong. Email: adamwong@ln.edu.hk. Phone: +852-26167149. Fax: +852-28917940.

1 Introduction

All-units discounts (AUDs) are a common vertical contract. Under AUDs, the per-unit price is cut on all units once the buyer’s purchase order reaches a threshold. The adoption of AUDs by dominant firms has become a prominent antitrust issue, such as in *Tomra*,¹ and in *Tetra Pak*.² In these cases, the dominant firms’ small rivals often only use linear pricing (LP). The offering of AUDs is deemed as a competition on the merits unless the undertaking is dominant in the market. So when small firms complain about AUDs offered by dominant undertakings as an abuse of dominance, one natural question to ask is: Why don’t small firms offer their own AUDs to compete when they can?

We find that such observed pricing asymmetry in aforementioned cases can be attributed to the small firm’s capacity constraint. The asymmetry in capacity between the firms, which gives rise to the captive market, allows the dominant firm to take advantage of the quantity commitment through all-units discounts while the capacity-constrained rival is induced to offer simple linear pricing.

We consider a duopoly model in which a full-capacity dominant firm competes with a capacity-constrained minor firm for a single buyer. To give the option of using AUDs to both firms and study which pricing policy they would choose in equilibrium, we consider the following four-stage game. In stage 0, each of the two firms simultaneously decides whether to commit itself to use LP or to use AUDs. The next two stages, stages 1 and 2, are pricing stages. In stage 1, each firm can either offer a pricing scheme (from the feasible set determined by its choice in stage 0), or wait until stage 2. In stage 2, any firm who chose to wait in stage 1 offers a pricing scheme (again from the feasible set determined by its choice in stage 0). In stage 3, the buyer chooses the quantities she purchases from the two firms.

Our main result (Theorem 1) is that *the asymmetry in capacity endogenously*

¹Case C-549/10P, *Tomra Systems and Others v. Commission*, Judgment of the Court (Third Chamber) of 19 April 2012.

²On November 16, 2016, the then State Administration of Industry and Commerce of China announced that, between 2009 and 2013, Tetra Pak abused its dominance in China’s aseptic packaging market. One of the abusive practices was Tetra Pak’s adoption of AUDs, which its small rivals do not use. For details on this case, see [Chao and Tan \(2017\)](#) and [Fu and Tan \(2019\)](#).

leads to asymmetries in the choices of pricing practices and the timing of making offers. More precisely, the game has a unique equilibrium outcome, in which only the minor firm commits itself to LP in stage 0, the dominant firm makes an AUDs offer in stage 1, and the minor firm waits in stage 1 and makes a LP offer in stage 2. [Chao, Tan, and Wong \(2018a\)](#), hereafter CTW) focus on a three-stage game in which a dominant firm offers AUDs first and then a capacity-constrained rival responds with LP, followed by a buyer’s purchase decision. The current paper shows that the exogenously assumed pricing and timing asymmetries in CTW will endogenously arise in equilibrium.

Intuitively, the minor firm restricts itself to LP upfront in order to soften the competition between the two firms. Knowing that the minor firm commits to LP, the dominant firm is encouraged to make its offer first (so that the minor firm can enjoy some second-mover benefits) and to make a less aggressive offer. This is the only way that the minor firm can make positive profits given the dominant firm can use AUDs. Such strategic competition-softening commitment can *only* be effectively used by the minor firm, not by the dominant firm. The reason hinges on two important features of our setting: *the asymmetry in capacity and the quantity threshold under AUDs*. Note that a sequential-move price competition in LP exhibits the features of first-mover disadvantage and second-mover advantage, since the follower can undercut the leader’s price. However, when the leader is allowed to use AUDs, the quantity threshold under AUDs introduces a quantity-strategic instrument into the price competition, so that the model now has some flavor of Stackelberg leadership game and hence can potentially exhibit a first-mover advantage from commitment value. As it turns out, whether a leader can enjoy such a Stackelberg-type first-mover advantage using AUDs depends on whether the leader has a captive demand, i.e., whether the follower is capacity-constrained. If the follower is the capacity-constrained firm, the price-undercutting threat from the follower would be limited and the first-mover advantage exists (Proposition 4). In contrast, if the follower is the unconstrained firm, the undercutting threat from the follower, even if it can only use LP, would be so severe that the first-mover advantage vanishes (Proposition 3). Consequently, by committing to LP, the minor firm can

induce the dominant firm to lead, but the dominant firm cannot induce the minor firm to lead by doing the same.

Our main result relies on the assumption that the minor firm is able to make an upfront commitment to use LP. However, the commitment need not be interpreted literally; it can be regarded as a reduced-form modeling that reflects development and maintenance of reputation or industry convention.³ The fundamental point made clear by the analysis of our four-stage game is that: if the minor firm can, by whatever means, commit upfront that it will only use LP, it would have incentives to do so; in contrast, the dominant firm has no such incentive to do so even if it could.

Related Literature. This paper is related to the oligopoly theory literature of endogenous order of moves. The most related one is [Hamilton and Slutsky \(1990\)](#). They generally proposed two extended games to endogenize the order of moves for a given “basic game,” termed the extended game with *observable delay* and the extended game with *action commitment*, respectively.⁴ Although we formally use “action commitment” to model the endogenous order of making offers, one can easily adapt our analysis to “observable delay” and show that the equilibrium outcome in our paper does not change if we use “observable delay” instead to model the pricing stages.⁵ However, we do not know of any paper about endogenous order that allows for nonlinear pricing, not to mention choices between linear pricing and nonlinear

³Even if the minor firm cannot commit in a binding way, in equilibrium it may commit in a non-binding but credible way once repeated interactions between the two firms are introduced. It could be because the minor firm has built a reputation of only using LP, or because it is a (non-binding) convention or culture of this industry that the minor firm only uses LP. The minor firm does not want to destroy the reputation or break the convention for long-term concerns. Both stories are consistent with our observations from the industries of interest that minor firms do not use AUDs.

⁴Under observable delay, before playing the basic game, players simultaneously commit whether to move early or late; if both choose “early” or both “late”, simultaneous moves follow; if one chooses “early” and the other “late”, sequential moves follow. Under action commitment, leadership means committing to a particular action (e.g., a price offer), whether or not the opponent attempts to lead or follow, like in the two pricing stages of our model.

⁵[van Damme and Hurkens \(2004\)](#) and [Amir and Stepanova \(2006\)](#) apply [Hamilton and Slutsky \(1990\)](#)’s extended game with observable delay and equilibrium selection based on risk dominance to show that the firm with a lower cost will become a leader. Their analyses are very different from ours.

pricing like in our paper.⁶

The efficiency analysis of our simultaneous-move subgames is related to the literature on competition in nonlinear pricing. When symmetric firms compete simultaneously in nonlinear pricing, [Bernheim and Whinston \(1986\)](#), [Bernheim and Whinston \(1998\)](#), and [O'Brien and Shaffer \(1997\)](#) show that the outcome is efficient under complete information. In our paper, the simultaneous price competition subgames in which the dominant firm can use AUDs (including simultaneous AUDs vs LP and simultaneous AUDs vs AUDs games) also lead to the efficient outcome (Proposition 1). However, other simultaneous price competition subgames in which the dominant firm can only use LP (including simultaneous LP vs LP and simultaneous LP vs AUDs subgames) result in inefficient outcomes (Proposition 2).⁷ Intuitively, the efficiency under simultaneous price competition relies on a firm who is both capacity-unconstrained and able to use nonlinear pricing to guarantee that no money would be left on the table.⁸

Recently, there have been some studies about competitive effects of AUDs. [Feess and Wohlschlegel \(2010\)](#) show that AUDs can shift the rent from the entrant to the coalition between the incumbent and the buyer. [Conlon and Mortimer \(2017\)](#) study the effects of AUDs used by a dominant chocolate candy manufacturer, and find empirical evidence that AUDs result in upstream foreclosure. [Chao, Tan, and Wong \(2018a\)](#) show that the AUDs adopted by a dominant firm can partially foreclose the capacity-constrained rival and harm the buyer.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 analyzes the equilibrium and states our results. Section 4 concludes. The proofs that are not provided in the text are in the Appendix.

⁶Closely related to the literature of endogenous order of moves, there is a strand of literature that deals with the determination of first- and second-mover advantages, e.g., [Gal-Or \(1985\)](#) and [Dowrick \(1986\)](#). We refer readers to [Amir and Stepanova \(2006\)](#) for an extended review of these two strands of literature.

⁷As we shall see, the simultaneous AUDs vs LP and simultaneous AUDs vs AUDs games are payoff-equivalent; the simultaneous LP vs LP and simultaneous LP vs AUDs games are payoff-equivalent.

⁸In our endogenous timing model, we do not have efficiency since in equilibrium, the second mover (firm 2) is capacity-constrained and restricts itself to LP.

2 Model

We consider a model of an intermediate goods market, in which there are two firms, firm 1 and firm 2, that produce identical products with zero marginal cost. There is one representative buyer (or downstream firm) for the product. The buyer can purchase from both firms. If she purchases $q_1 \geq 0$ units from firm 1 and $q_2 \geq 0$ units from firm 2, her payoff is the gross benefit (or revenue from reselling to final consumers) given by $u(q_1 + q_2)$, less the payments to the two firms.

2.1 Demand and Asymmetry in Capacity

The buyer's gross benefit function $u(q)$ is twice continuously differentiable, increasing, and strictly concave for any quantity below the welfare-maximizing quantity q^e , where $u'(q^e) = 0$ and $0 < q^e < \infty$, and $u(0) = 0$. Let the optimal quantity demanded by the buyer at per-unit price p be $D(p) \equiv \operatorname{argmax}_{q \geq 0} \{u(q) - p \cdot q\}$. With the concavity of $u(\cdot)$, $D(p)$ exists and is uniquely determined by $u'(D(p)) = p$ for $0 \leq p \leq u'(0)$. In particular, $q^e = D(0)$.

We assume that demand $D(\cdot)$ is concave on $[0, u'(0)]$. It follows that the monopoly profit function $\pi(p) \equiv p \cdot D(p)$ is strictly concave in p on $[0, u'(0)]$.⁹ Denote $p^m \equiv \operatorname{argmax}_p \pi(p)$ as the monopoly price.

Firm 1 is a dominant firm and firm 2 a minor firm in the following sense. Firm 1 has full capacity to serve the whole demand of the buyer, whereas firm 2 is capacity-constrained in the sense that it can produce up to its capacity k . We assume that, firm 2's capacity level is strictly less than the socially efficient level of quantities, i.e., $0 < k < q^e$, implying that firm 2 cannot serve the whole demand of the buyer when two firms compete à la Bertrand.

We call

$$D^{cap}(\cdot) \equiv \max\{D(\cdot) - k, 0\}$$

⁹The only place where we need the assumption of concave demand (rather than a weaker one such as log-concave demand or strictly concave monopoly profit) is the establishment of Proposition 4, which refers to CTW who assume concave demand. However, the main message of the current paper remains valid even if we relax the concavity of demand. See footnote 17 for the details.

firm 1's *captive demand function*. Intuitively, from firm 1's point of view, this portion of the total demand is not subject to any threat of competition from firm 2, due to the latter's capacity constraint.

It is useful to do a thought experiment as follows: consider that the buyer always buys her first k units from firm 2 so that firm 1 is simply a monopolist over its captive demand. Then, if firm 1 can only use linear pricing, its profit-maximizing price is

$$p^{cap} \equiv \operatorname{argmax}_p p \cdot D^{cap}(p),$$

which is determined by

$$\pi'(p^{cap}) = k,$$

and the corresponding profit is

$$\pi^{cap} \equiv p^{cap} D^{cap}(p^{cap}) = \pi(p^{cap}) - p^{cap} k.$$

On the other hand, if firm 1 can use nonlinear pricing (e.g., AUDs below), it would be able to capture the efficient surplus

$$S^{cap} \equiv u(q^e) - u(k),$$

corresponding to the captive demand.

One can readily verify that $0 < p^{cap} < \min\{p^m, u'(k)\}$, and $\pi^{cap} < S^{cap}$.

2.2 Linear Pricing (LP) and All-Units Discounts (AUDs)

The complexity of the pricing schemes for each firm will be endogenously determined in our model. More precisely, each firm can commit upfront to the class of pricing schemes from which it may choose to offer in the price competition between the two firms. We consider two classes of pricing schemes. One is of the form *linear pricing (LP)*: a pricing scheme of the LP form is simply a single unit price $p \geq 0$.

The other is of the form *all-units discounts (AUDs)*: a pricing scheme of AUDs consists of a triple (r, Q, p) with $r \geq p \geq 0$ and $Q \geq 0$. Here r is the per-unit price

when the quantity purchased is less than the quantity threshold Q , and p is the per-unit price for all units once the quantity purchased reaches Q . In other words, the total payment for purchasing q units under AUDs (r, Q, p) is

$$\begin{cases} r \cdot q & \text{if } q < Q \\ p \cdot q & \text{if } q \geq Q \end{cases}.$$

Denote the set of all AUDs schemes as \mathcal{T}^A , and the set of all LP schemes as \mathcal{T}^L .

Although AUDs are more general than LP, as we will see, a firm may still want to strategically commit upfront to use the simple LP due to the following two competition-softening concerns. First, by committing upfront to LP, a firm may be able to induce the other firm to make offers less aggressively. Second, in our model the order of making offers is endogenously determined. By committing upfront to LP, a firm may be able to induce the other firm to make offers first, and thus enjoy the second-mover advantage.

2.3 The Game

In the price competition between the two firms, both the combination of pricing practices (i.e., AUDs vs AUDs, or AUDs vs LP, or ...) and the order of making offers (i.e., whether the firms make offers simultaneously or sequentially; if sequentially, who is the leader and who is the follower) are endogenous. More precisely, we consider the following four-stage game, denoted as Γ , in which the actions in each stage are simultaneously taken, and all actions taken in earlier stages are observable by all players.

Stage 0 (pre-commitment stage): Each firm $i \in \{1, 2\}$ chooses a set of feasible pricing schemes (also known as tariff space) $\mathcal{T}_i \in \{\mathcal{T}^A, \mathcal{T}^L\}$.

Stage 1 (pricing stage 1): Each firm i chooses an action from $\mathcal{T}_i \cup \{\text{“Wait”}\}$, i.e., either to offer a pricing scheme $\tau_i \in \mathcal{T}_i$ or to wait.

Stage 2 (pricing stage 2): Each firm i who chose “Wait” in Stage 1 chooses its pricing scheme $\tau_i \in \mathcal{T}_i$.

Stage 3 (purchasing stage): The buyer chooses quantities $q_1 \in \mathbb{R}_+$ and $q_2 \in [0, k]$ she buys from firms 1 and 2, respectively.

Note that the only fundamental asymmetry between the two firms is that firm 2 is capacity-constrained whereas firm 1 is not. We will illustrate that the asymmetry in capacity endogenously generates the asymmetries in pricing practices and order of making offers between the two firms.

Since Γ is a multi-stage game with complete and perfect information, we adopt the equilibrium concept of subgame-perfect equilibrium (SPE).¹⁰ However, Γ has multiple SPE outcomes and some of them are not as reasonable as others. Therefore, we will select SPEs based on some refinements that we explain below. It turns out that our refinements allow us to make a nice unique prediction.

In each step along the backward induction analysis of our four-stage game Γ , we are solving some simultaneous-move game, which we hope to have a unique pure-strategy Nash equilibrium. Unfortunately, some of the subgames have no pure-strategy equilibrium. For example, for the subgame after $(\mathcal{T}^L, \mathcal{T}^L)$ has been chosen in stage 0 and (“Wait”, “Wait”) has been chosen in stage 1 (so that the subgame is the simultaneous-move LP vs LP game with a capacity-constrained firm), it is well known that no pure-strategy equilibrium exists.¹¹ Therefore, we have to allow for mixed strategies. With that being said, it is a common practice in applied literature (sometimes even without explicit mentioning) to focus on pure-strategy equilibria whenever such equilibria exist. We follow this practice and select only pure-strategy equilibria whenever possible, in each step of the backward induction procedure.¹²

¹⁰Since the tariff spaces and the order of making offers vary across different subgames, there is not a single tie-breaking rule of the buyer that is “sensible” in all subgames, in the sense that it is consistent with the existence of equilibrium. In our analysis, the tie-breaking rule for each particular subgame is endogenously determined such that both firms have no profitable deviation. For example, if the two firms make offers sequentially, the buyer breaks the tie in favor of the follower; in the simultaneous AUDs vs AUDs/LP games, the buyer breaks the tie in favor of firm 1.

¹¹The mixed-strategy equilibria of such Bertrand competition games with capacity constraints are characterized by [Kreps and Scheinkman \(1983\)](#).

¹²This selection criterion is important for our analyses of the AUDs vs LP and the AUDs vs AUDs subgames, which require characterizing the equilibrium profits for the corresponding simultaneous-move games. Note that Proposition 1 below only characterizes the *pure-strategy* SPE

Moreover, it will become clear in Subsection 3.2 that, when we analyze stage 1 along the backward induction procedure, there are more than one pure-strategy Nash equilibria for some subgames. However, in those reduced subgames there is a natural way to select equilibrium: elimination of weakly dominated strategies. In particular, in those reduced subgames we will see that “Wait” is the weakly dominant strategy for one or both firms.

To sum up, we select equilibria in each step of the backward induction procedure by the following two criteria: (1) select pure-strategy equilibria whenever such equilibria exist; (2) select those pure-strategy equilibria that do not involve weakly dominated strategies whenever such equilibria exist. We call any SPE that survives the above two selection criteria a *refined SPE*. We will show that our four-stage game Γ has a unique refined SPE outcome.

3 Equilibrium Analysis

Along the way to solve the four-stage game Γ , we need to consider many subgames. The choices made in stage 0 generate four possible combinations of tariff spaces: AUDs vs AUDs, AUDs vs LP, LP vs AUDs, and LP vs LP. Moreover, according to the timing choices made in stages 1 and 2, the offers of the two firms may be made simultaneously or sequentially.

To introduce convenient notations, let $\Pi_i^{\{AA\}}, \Pi_i^{\{AL\}}, \Pi_i^{\{LA\}}, \Pi_i^{\{LL\}}$ denote firm i 's equilibrium (expected) profit if the two firms make simultaneous offers according to the tariff spaces indicated in the superscript. That is, e.g., $\Pi_1^{\{AL\}}$ is firm 1's equilibrium profit in an auxiliary model where firm 1 offers AUDs, firm 2 offers LP, and the two offers are made simultaneously. Also, let $\Pi_i^{A_1A_2}, \Pi_i^{A_1L_2}, \Pi_i^{L_1A_2}, \Pi_i^{L_1L_2}$ denote firm i 's equilibrium profit if the two firms make sequential offers according to the tariff spaces and order indicated in the superscript. That is, e.g., $\Pi_2^{A_1L_2}$ is firm

profits for the simultaneous AUDs vs LP and AUDs vs AUDs games. We conjecture that there are not mixed-strategy SPE for those games leading to other profits, but a rigorous proof would be technically involved. The selection criterion here makes this conjecture irrelevant and its proof unnecessary.

2's equilibrium profit in an auxiliary model where firm 1 offers AUDs first, and then firm 2 offers LP. Analogously, $\Pi_i^{A_2A_1}, \Pi_i^{A_2L_1}, \Pi_i^{L_2A_1}, \Pi_i^{L_2L_1}$ are the equilibrium profits when firm 2 makes its offer first.

Consider the subgames starting from Stage 2. Fix any combination of tariff spaces $\mathcal{T}_1, \mathcal{T}_2 \in \{\mathcal{T}^A, \mathcal{T}^L\}$ chosen in Stage 0. If in Stage 1 firm j has made its offer $\tau_j \in \mathcal{T}_j$ but firm i has chosen "Wait", then firm i in stage 2 would offer a best response to τ_j in \mathcal{T}_i . As can be seen later, for our purpose we do not need to characterize firm i 's best response in \mathcal{T}_i to any given τ_j . For now we only need to observe that the continuation subgame in which firm i and the buyer sequentially take actions has some SPE. Indeed, the buyer's optimal purchases always exist (even if we restrict attention to pure strategies);¹³ if we let the buyer use a tie-breaking rule in favor of firm i , clearly firm i 's best response exists.

If both firms have made their offers $\tau_1 \in \mathcal{T}_1$ and $\tau_2 \in \mathcal{T}_2$ in Stage 1, then there are no actions to be taken in Stage 2 and the buyer simply chooses her optimal purchases in Stage 3.

In the following subsection, we consider the remaining possibility for the subgames starting from Stage 2, i.e., both firms have chosen "Wait" in Stage 1.

3.1 Simultaneous Offers

If $\mathcal{T}_1, \mathcal{T}_2 \in \{\mathcal{T}^A, \mathcal{T}^L\}$ have been chosen in Stage 0, and both firms have chosen "Wait" in Stage 1, then the continuation subgame starting from Stage 2 is the simultaneous-move (for the two firms) version of the \mathcal{T}_1 vs \mathcal{T}_2 price competition game. This subsection characterizes the refined SPE profits under simultaneous moves for every $\mathcal{T}_1, \mathcal{T}_2 \in \{\mathcal{T}^A, \mathcal{T}^L\}$.

We start with the two simultaneous-move games with $\mathcal{T}_1 = \mathcal{T}^A$.

Proposition 1 (Simultaneous AUDs vs LP/AUDs). *Both the simultaneous-move version of the AUDs vs LP game and the simultaneous-move*

¹³That the buyer's optimal purchases always exist is from a familiar argument, whose main elements are (1) $u(\cdot)$ is continuous and $q^e < \infty$, and (2) both AUDs and LP are lower semi-continuous functions.

version of the AUDs vs AUDs game have a unique pure-strategy SPE payoff allocation, in which firm 1's profit is $\Pi_1^{\{AL\}} = \Pi_1^{\{AA\}} = S^{cap}$, firm 2's profit is $\Pi_2^{\{AL\}} = \Pi_2^{\{AA\}} = 0$, and the buyer's surplus is $u(k)$.

Proposition 1 essentially says that for the two simultaneous-move games where firm 1 can use AUDs, in equilibrium, firm 2 earns zero profit, firm 1 gets the highest possible profit S^{cap} from its captive demand, and the buyer gets the highest possible surplus $u(k)$ that firm 2 and the buyer can jointly generate. (It implies efficiency in the sense that the equilibrium total payoff is $u(q^e)$.)

For these two games, the following is an equilibrium outcome: firm 1 offers the AUDs scheme characterized by $(r_1, Q_1, p_1) = (\infty, q^e, S^{cap}/q^e)$, firm 2 offers the LP scheme with price $p_2 = 0$, and the buyer purchases q^e from firm 1 and 0 from firm 2. By setting the quantity threshold q^e and a prohibitively high pre-threshold price, firm 1 is effectively offering an exclusive contract: the buyer has to either buy nothing from firm 1 (and hence single-source from firm 2), or buy the efficient quantity q^e from firm 1 (and hence buy nothing from firm 2). In equilibrium the buyer must accept the contract.

To see that the above is an equilibrium outcome, first note that firm 1 and the buyer are splitting the total surplus $u(q^e)$ in a way that the buyer is indifferent between accepting and rejecting the contract given firm 2's price $p_2 = 0$, i.e., buyer's surplus is $u(k)$ and firm 1's profit is $u(q^e) - u(k) = S^{cap}$. Given $p_2 = 0$, clearly firm 1 can only extract surplus from its captive demand and thus S^{cap} is its highest possible profit. On the other hand, although firm 2 makes zero profit, it has no profitable deviation *even if it can use AUDs*: by accepting firm 1's exclusive contract, the buyer gets surplus $u(k)$ from purchasing q^e from firm 1 and 0 from firm 2; given this, firm 2 has no hope to, without making loss, induce the buyer to reject firm 1's contract, because the joint surplus of firm 2 and the buyer (without firm 1) must not be higher than $u(k)$.

An important implication of Proposition 1 is that the two simultaneous-move games where firm 1 can use AUDs are payoff-equivalent. In particular, provided firm 1 can use AUDs, being able to use AUDs does not help firm 2.

Now we turn to the other two simultaneous-move games with $\mathcal{T}_1 = \mathcal{T}^L$. The

simultaneous LP vs LP game is essentially a special case of the price competition subgame in [Kreps and Scheinkman \(1983\)](#) (with only one firm being capacity constrained). It is well known that this version of the price competition game has no pure-strategy equilibrium. Before characterizing the expected profits in mixed-strategy equilibria, it will prove informative to first consider the so-called “optimal leader’s prices” and the “optimal leader’s profits” under the situation in which one firm makes its LP offer first.

Consider that $\mathcal{T}_1 = \mathcal{T}_2 = \mathcal{T}^L$ and firm 1 has to make its offer first. Then it is straightforward to see that (since firm 2 always wants to undercut firm 1’s price provided firm 1’s price is positive) firm 1 is effectively facing only its captive demand. Consequently, firm 1’s optimal leader’s price is $p_1^{L_1L_2} = p^{cap}$ and its optimal leader’s profit is $\Pi_1^{L_1L_2} = \pi^{cap}$.

Consider that $\mathcal{T}_1 = \mathcal{T}_2 = \mathcal{T}^L$ and firm 2 has to make its offer first. Then firm 2 must choose its offer subject to the constraint that firm 1 does not have an incentive to undercut its offer. That is, firm 2 seeks a p_2 to solve

$$\begin{aligned} & \underset{p_2}{\text{Maximize}} [p_2 \cdot \min\{D(p_2), k\}] \\ & \text{s.t. } \pi(p_1) \leq \pi^{cap} \text{ for every } p_1 \in [0, p_2]. \end{aligned}$$

Note that the above constraint must be binding. Therefore, firm 2’s optimal leader’s price $p_2^{L_2L_1}$ is the unique \underline{p} satisfying $\pi(\underline{p}) = \pi^{cap}$ and $\underline{p} < p^{cap}$. It follows that $0 < p_2^{L_2L_1} < u'(k)$, and therefore firm 2’s optimal leader’s profit is

$$\Pi_2^{L_2L_1} = p_2^{L_2L_1} k > 0.$$

As [Kreps and Scheinkman \(1983\)](#) show, under simultaneous LP vs LP, the two firms’ equilibrium expected profits are their optimal leader’s profits, and the equilibrium price support is the interval between the two optimal leader’s prices, i.e., $[p_2^{L_2L_1}, p^{cap}]$. We find that these also hold for the simultaneous LP vs AUDs game.¹⁴

¹⁴The proof of Proposition 2 actually reveals that the two simultaneous-move games where firm 1 can only use LP are outcome-equivalent.

Proposition 2 (Simultaneous LP vs LP/AUDs). *Both the simultaneous-move version of the LP vs LP game and the simultaneous-move version of the LP vs AUDs game do not have pure-strategy SPE but have at least one mixed-strategy SPE. In any mixed-strategy SPE, firm 1's expected profit is $\Pi_1^{\{LL\}} = \Pi_1^{\{LA\}} = \Pi_1^{L_1L_2} = \pi^{cap} > 0$ and firm 2's expected profit is $\Pi_2^{\{LL\}} = \Pi_2^{\{LA\}} = \Pi_2^{L_2L_1} = p_2^{L_2L_1}k > 0$.*

Proposition 2 essentially says that for the two simultaneous-move games where firm 1 can only use LP, in equilibrium each firm earns an expected profit equal to its optimal leader's profit (i.e., the maximum profit it could earn if it were the first mover). In particular, provided firm 1 can use only LP, being able to use AUDs does not help firm 2.¹⁵

We have presented our results for the four subgames with simultaneous offers. The resulting equilibrium profits (from Propositions 1 and 2) are summarized in Table 1. In particular, for firm 2, being able to use AUDs does not help, no matter whether firm 1 can or cannot use AUDs; in contrast, for firm 1, being able to use AUDs always helps, no matter whether firm 2 can or cannot use AUDs. Fundamentally, it is due to the fact that *firm 1 has a downward-sloping captive demand, from which AUDs can help extract surplus, while firm 2 does not.*

		Firm 2	
		\mathcal{T}^A	\mathcal{T}^L
Firm 1	\mathcal{T}^A	$S^{cap}, 0$	$S^{cap}, 0$
	\mathcal{T}^L	$\pi^{cap}, \Pi_2^{L_2L_1}$	$\pi^{cap}, \Pi_2^{L_2L_1}$

Table 1: Profits under Simultaneous Offers

It is clear from Table 1 that, if the order of making offers were exogenously assumed to be simultaneous (i.e., if Stage 1 were eliminated in Γ), then firm 1 would choose the AUDs tariff space in Stage 0, but we could not conclude on firm 2's choice of tariff space. The next question is: what if the order of making offers is endogenous? So we turn to the analysis for Stage 1 of Γ .

¹⁵For more intuition about the simultaneous LP vs LP and LP vs AUDs games, see our working paper version [Chao, Tan, and Wong \(2018b\)](#).

3.2 Order of Making Offers

We now consider the four subgames starting from Stage 1 (where the order of making offers is endogenous), which we call AUDs vs AUDs subgame, AUDs vs LP subgame, and so on. The following proposition shows that, in Stage 1, making an offer is weakly dominated by “Wait” at least for firm 2, and usually also for firm 1. This is where our second criterion of equilibrium selection is important. If we do not eliminate weakly dominated strategies, in the AUDs vs AUDs and AUDs vs LP subgames, both firms might make offers in Stage 1 (which is weakly dominated). Indeed, in SPE the two firms can offer in Stage 1 what they would offer in a pure-strategy SPE of the corresponding simultaneous-move version game. Given that the other firm does that, each firm cannot gain by deviating to wait. (Of course, these SPEs fail to be refined SPE.)

Proposition 3. *For the AUDs vs AUDs, LP vs LP, and LP vs AUDs subgames of Γ , after reducing the subgames starting from Stage 2, choosing “Wait” in Stage 1 is both firms’ weakly dominant strategy in the reduced game. For the AUDs vs LP subgame of Γ , after reducing the subgames starting from Stage 2, choosing “Wait” in Stage 1 is firm 2’s weakly dominant strategy in the reduced game.*

Here we briefly explain why, in Stage 1, “Wait” is usually weakly dominant. Consider firm i in Stage 1. Let us compare “Wait” with a particular offer $\tau_i \in \mathcal{T}_i$ for firm i . If the rival firm $j \neq i$ makes some offer $\tau_j \in \mathcal{T}_j$ in Stage 1, then for firm i it is always weakly better to wait and then in Stage 2 make an offer that is a best response to τ_j (and it is strictly better to wait whenever τ_i is not a best response to τ_j). On the other hand, if the rival firm j waits in Stage 1, then, by choosing “Wait” firm i would earn its simultaneous-move equilibrium profit, and by offering τ_i firm i ’s profit would not be higher than its optimal leader’s profit. In most of the cases, firm i ’s optimal leader’s profit is not higher than its simultaneous-move equilibrium profit, because of the follower’s threat of undercutting. It turns out that the above is true except in only one case: $i = 1$ and $\mathcal{T}_i = \mathcal{T}^A$ and $\mathcal{T}_j = \mathcal{T}^L$, that is, except when firm i is the dominant firm, who is able to use AUDs and facing a minor firm who can only use LP.

From the first part of Proposition 3, the AUDs vs AUDs, LP vs LP, and LP vs AUDs subgames of Γ all reduce to the corresponding simultaneous-move games (both firms wait in Stage 1 and simultaneously make offers in Stage 2 in any refined SPE). The resulting refined SPE profits in those subgames are given by Propositions 1 and 2, which we reproduce in Table 2.

		Firm 2	
		\mathcal{T}^A	\mathcal{T}^L
Firm 1	\mathcal{T}^A	$S^{cap}, 0$	$\Pi_1^{A_1L_2}, \Pi_2^{A_1L_2}$
	\mathcal{T}^L	$\pi^{cap}, \Pi_2^{L_2L_1}$	$\pi^{cap}, \Pi_2^{L_2L_1}$

Table 2: Normal Form of Game Γ at Stage 0

For the AUDs vs LP subgame of Γ , the second part of Proposition 3 says that, in any refined SPE, firm 2 waits in Stage 1. The remaining question is: with the belief that firm 2 would wait in Stage 1 in this subgame, does firm 1 have incentive to be a leader (i.e., to make an AUDs offer in Stage 1)? The answer turns out to be “yes.” It is due to the analysis of CTW, which shows that, given that the minor firm is the follower and can only use LP, the dominant firm can use AUDs to make higher profits than S^{cap} . Intuitively, AUDs allow the dominant firm to leverage its market power from its captive portion of demand to the competitive portion, and at the same time, prevent its minor rival from undercutting.¹⁶ In addition, CTW also illustrate that it is optimal for the dominant firm to partially instead of fully foreclosing its minor rival, so that in equilibrium the minor firm also makes positive profits.

Proposition 4. *The AUDs vs LP subgame of Γ has a unique refined SPE outcome, in which*

¹⁶Here the LP restriction of firm 2 is important. If firm 2, as a follower, can use AUDs, then firm 1, as a leader, cannot make higher profits than S^{cap} . To see this, suppose on the contrary that firm 1’s profit exceeds S^{cap} . Then, noting that the largest feasible total surplus is $u(q^e)$ and recalling that $u(k) = u(q^e) - S^{cap}$, the total payoff of firm 2 and the buyer would be strictly below $u(k)$, but then firm 2 would be able to make some AUDs offer with threshold k to ensure the sum of firm 2’s profit and the buyer’s surplus to be $u(k)$ and both parties to be strictly better off; thus firm 2 would have a profitable deviation, which is a contradiction.

- (a) firm 1 makes its offer in Stage 1 and firm 2 makes its offer in Stage 2,
- (b) the buyer's purchases (q_1, q_2) satisfy $q^e - k < q_1 < q^e$, $0 < q_2 < k$, and $q_1 + q_2 < q^e$,
- (c) firm 1's profit is $\Pi_1^{A_1L_2}$ and firm 2's profit is $\Pi_2^{A_1L_2}$, satisfying

$$\Pi_1^{A_1L_2} > S^{cap}, \text{ and } \Pi_2^{A_1L_2} > 0.$$

From Proposition 4(c) the refined SPE profits for the AUDs vs LP subgame of Γ are as shown in Table 2.

3.3 Solving the Whole Game

To solve Γ , it remains to determine the choices of tariff spaces in Stage 0, i.e., solve the reduced game illustrated in Table 2.

It is easy to see from our previous analysis that this reduced game can be solved by iterated elimination of strictly dominated strategies. First, for firm 1, \mathcal{T}^A strictly dominates \mathcal{T}^L because $\Pi_1^{A_1L_2} > S^{cap} > \pi^{cap}$ according to Proposition 4(c). Second, given that firm 1 chooses \mathcal{T}^A , for firm 2, \mathcal{T}^L strictly dominates \mathcal{T}^A because $\Pi_2^{A_1L_2} > 0$, again according to Proposition 4(c). The unique Nash equilibrium of the reduced game in Table 2 is $(\mathcal{T}^A, \mathcal{T}^L)$. This together with Proposition 4 implies the following main theorem.¹⁷

Theorem 1. *Game Γ has a unique refined SPE outcome, in which*

- (a) firm 1 chooses $\mathcal{T}_1 = \mathcal{T}^A$ and firm 2 chooses $\mathcal{T}_2 = \mathcal{T}^L$ in Stage 0,
- (b) firm 1 makes its AUDs offer in Stage 1, firm 2 waits in Stage 1 and makes its LP offer in Stage 2,

¹⁷In CTW, the assumption of concave demand is needed to ensure that firm 1's profit, after incorporating firm 2's best response, is single-peaked; this in turn ensures the uniqueness of equilibrium and facilitates comparative static analysis. If we relax the concavity of demand, the AUDs vs LP subgame might have multiple equilibrium outcomes. However, in every equilibrium for the subgame firm 1's and firm 2's profits must still exceed S^{cap} and 0, respectively. As a result, although the whole four-stage game Γ might then have multiple refined SPE outcomes, the main message of the paper remains valid: in every refined SPE, the dominant firm chooses to adopt AUDs and be a leader, and the minor firm chooses to adopt LP and be a follower.

(c) the buyer's purchases (q_1, q_2) satisfy $q^e - k < q_1 < q^e$, $0 < q_2 < k$, and $q_1 + q_2 < q^e$,

(d) firm 1's profit is strictly higher than S^{cap} and firm 2's profit is positive.

In the unique refined SPE outcome, only firm 1 uses AUDs, while firm 2 restricts itself to LP. The two firms make offers sequentially, with firm 1 being the leader. Firm 1 supplies more than its captive portion of demand and earns more than the full surplus from its captive demand. Firm 2 makes positive profits by supplying positive quantity, yet less than its capacity level. Their total output is lower than the efficient level.¹⁸

To understand why firm 2 restricts itself to LP, it is important to understand the role of the endogenous order of making offers in our analysis. If the order is exogenously assumed to be simultaneous, our model reduces to the normal form game illustrated in Table 1. Then Firm 1 chooses AUDs. Under simultaneous AUDs vs AUDs or simultaneous AUDs vs LP, the price competition between the two firms is so severe that firm 1's equilibrium profit is only the full surplus S^{cap} from its captive demand, and firm 2's equilibrium profit is 0 regardless of its tariff space since it does not have captive demand. Therefore firm 2 has no incentive to restrict itself to LP.

In contrast, under our endogenous order, our model reduces to the normal form game illustrated in Table 2. Moving from Table 1 to Table 2 amounts to raising the profit pair under $(\mathcal{T}^A, \mathcal{T}^L)$ from $(S^{cap}, 0)$ to $(\Pi_1^{A_1L_2}, \Pi_2^{A_1L_2})$, which strictly exceeds $(S^{cap}, 0)$. This gives firm 2 the incentive to restrict itself to LP. Firm 2 understands that, if it can make firm 1 believe that it will only use LP after knowing firm 1's offer, then firm 1 has incentive to lead.¹⁹ That is, under endogenous order, firm 2 turns a simultaneous-move price competition into a sequential-move one by restricting to LP. Consequently, the price competition is softened, implying higher profits for both

¹⁸Since the equilibrium outcome here is the same as the one in CTW, we refer interested readers to consult CTW for the characterization of the equilibrium offers and the corresponding welfare analysis and comparative statics.

¹⁹In the model we simply assume that firm 2 can commit upfront. But it would work equally well if firm 2 can, by whatever means, make firm 1 believe that it will only use LP after knowing firm 1's offer. Reputation concern or industry culture provides realistic justifications for such a means. See footnote 3.

firms, at the expenses of the buyer and aggregate efficiency.

4 Conclusion

In this paper, we investigate the strategic choice of whether to offer AUDs or LP, as well as whether to lead or to follow, in a duopoly game where firms differ in their capacity level. We find that, only the dominant firm (with full capacity) has incentives to adopt the AUDs and be a leader, whereas the minor firm (with capacity constraint) chooses to restrict itself to LP and be a follower. That is, given the exogenous asymmetry in capacity, the two firms endogenously differentiate between themselves in terms of tariff spaces and timing of making offers. The asymmetries in capacity, tariff spaces, and timing of making offers together allow the firms to soften price competition, through the leader's quantity threshold based pricing schemes and the follower's capacity constraint.

Our analysis provides a plausible explanation for a puzzle arising in many antitrust cases involving AUDs: why don't small rivals adopt the AUDs as the dominant firms do? We suggest that the asymmetry in capacity between the firms (or more generally be interpreted as factors such as distribution channels or ability to access buyers), which gives rise to the captive market, allows the dominant firm to take advantage of the quantity commitment through AUDs while the capacity-constrained rival is induced to offer simple tariffs. Our setting is consistent with many of the basic features that characterize the industries in some of the recent antitrust cases.

Although we focus on AUDs in our analysis due to the original motivations from a number of antitrust cases, we expect that our approach applies to other complex pricing schemes, such as quantity forcing, three-part tariffs, or even general nonlinear pricing, and our main results still hold. Besides, we focus on the case with a single downstream buyer. If there are multiple buyers, then the dominant firm's optimization program would be much more complex because its formulation would depend on (1) whether the dominant firm may offer a personalized AUDs for each buyer, or a common AUDs tariff to all buyers, and (2) when the minor firm's capacity

falls short of the total desired purchase from all buyers, how the minor firm rations among the buyers' demands. We leave these issues to future research.

Appendix

Proof of Proposition 1. The existence of pure-strategy SPE has been shown in the main text. In the following, consider any pure-strategy SPE of a simultaneous-move \mathcal{T}^A vs \mathcal{T}_2 game, where $\mathcal{T}_2 \in \{\mathcal{T}^A, \mathcal{T}^L\}$. Let π_i denote firm i 's equilibrium profit and BS denote the buyer's equilibrium surplus. We must show $(\pi_1, \pi_2, BS) = (S^{cap}, 0, u(k))$.

Step 1. $\pi_1 + \pi_2 + BS \leq u(q^e)$. It is because the largest feasible total surplus is $u(q^e)$.

Step 2. $\pi_2 = 0$. Clearly, $\pi_2 < 0$ is impossible in equilibrium. Suppose $\pi_2 > 0$. Then, from Step 1 we have $\pi_1 + BS < u(q^e)$. But then, given firm 2's offer, firm 1 is able to make some AUDs offer with threshold q^e to ensure the sum of firm 1's profit and the buyer's surplus to be $u(q^e)$ and both parties to be strictly better off. Therefore, firm 1 has a profitable deviation, a contradiction.

Step 3. $\pi_1 \geq S^{cap}$. It is because firm 1 can guarantee itself a profit arbitrarily close to S^{cap} by offering the AUDs scheme characterized by $(r_1, Q_1, p_1) = (\infty, q^e, (S^{cap} - \varepsilon)/q^e)$ with $\varepsilon > 0$. Indeed, if firm 1 offers this AUDs scheme, the buyer would purchase q^e units from firm 1 regardless of firm 2's offer. (Note that the resulting firm 1's profit is $S^{cap} - \varepsilon$ and buyer's surplus is $u(k) + \varepsilon$.)

Step 4. $BS \geq u(k)$. Suppose on the contrary that $BS < u(k)$. Then, firm 2 is able to make positive profits by offering a LP scheme, contradicting Step 2. Indeed, if firm 2 sets a price $p_2 > 0$ such that $u(k) - p_2 k > BS$, then the buyer would buy a positive quantity from firm 2 (since the buyer's surplus would be at most BS if the buyer buys nothing from firm 2).

Combining the above steps and recalling $S^{cap} + u(k) = u(q^e)$, we have $(\pi_1, \pi_2, BS) = (S^{cap}, 0, u(k))$. ■

The proof of Proposition 2 requires the following lemma.

Lemma A.1. *In any (pure-strategy or not) SPE of the simultaneous-move version of the LP vs LP game or the simultaneous-move version of the LP vs AUDs game, firm 1's expected profit is at least $\pi^{cap} > 0$, firm 2's expected profit is positive, and*

the probability that firm 2 sells k units is positive.

Proof. Consider any (pure-strategy or not) SPE of the simultaneous-move \mathcal{T}_1 vs \mathcal{T}_2 game, where $\mathcal{T}_1 = \mathcal{T}^L$ and $\mathcal{T}_2 \in \{\mathcal{T}^A, \mathcal{T}^L\}$. Let $\Psi_1(\cdot)$ denote the equilibrium c.d.f. of firm 1's offer. Let \bar{p}_1 and \underline{p}_1 denote the supremum and infimum of the support of $\Psi_1(\cdot)$. Let p_1 denote firm 1's offer, which is a random variable distributed according to $\Psi_1(\cdot)$. Let π_i denote firm i 's expected profit in the equilibrium. First, if firm 1 offers p^{cap} , its profit is at least π^{cap} , regardless of what firm 2 offers. Therefore, $\pi_1 \geq \pi^{cap} > 0$. It follows that $\underline{p}_1 > 0$. Then firm 2 is able to make positive profit by offering, for example, a linear pricing with per-unit price above 0 and below \underline{p}_1 . Therefore, $\pi_2 > 0$.

Let q_2 denote firm 2's equilibrium sales, which is a random variable. It remains to prove $\Pr(q_2 = k) > 0$. We first consider the $\mathcal{T}_2 = \mathcal{T}^L$ case. For this case, let \bar{p}_2 denote the supremum of the support of firm 2's equilibrium price p_2 . If $\bar{p}_1 < \bar{p}_2$, then $\pi_2 = 0$ (since firm 2 would sell nothing by making any offer in the neighborhood of \bar{p}_2), contradicting our previous result. Similarly, if $\bar{p}_1 = \bar{p}_2$ and $\Pr(p_1 = \bar{p}_1) = 0$, then $\pi_2 = 0$, a contradiction. Therefore, we must have either $\bar{p}_1 > \bar{p}_2$, or $\bar{p}_1 = \bar{p}_2$ and $\Pr(p_1 = \bar{p}_1) > 0$. If $\bar{p}_1 > \bar{p}_2$, then we must have $\bar{p}_1 \leq u'(k)$, for otherwise $\pi_1 = 0$ (since firm 1 would sell nothing by making any offer in the neighborhood of \bar{p}_1), contradicting our previous result; then $\Pr(q_2 = k) \geq \Pr(p_1 > \bar{p}_2) > 0$ follows from $\bar{p}_2 < \bar{p}_1 \leq u'(k)$, and we are done. Now suppose $\bar{p}_1 = \bar{p}_2 \equiv \bar{p} > 0$ and $\Pr(p_1 = \bar{p}) > 0$. Then $\Pr(p_2 = \bar{p}) = 0$, for otherwise it would be a positive probability event that both firms offer \bar{p} , but then at least one firm would strictly prefer offering somewhere below \bar{p} to offering \bar{p} , no matter how we let the buyer break the tie when both firms offer \bar{p} , which is a contradiction. We again must have $\bar{p} \leq u'(k)$, for otherwise $\pi_1 = 0$, a contradiction. It follows that $\Pr(q_2 = k) \geq \Pr(p_1 = \bar{p}) > 0$ and we are done.

In the rest of the proof, we consider the $\mathcal{T}_2 = \mathcal{T}^A$ case. Let $\hat{\tau}_2 \in \mathcal{T}^A$ be a firm 2's pure-strategy best response to $\Psi_1(\cdot)$ when the buyer uses some tie-breaking rule $\hat{\beta}$. Such a best response exists for some buyer's tie-breaking rule, because $\Psi_1(\cdot)$ is firm 1's equilibrium strategy. Since $\hat{\tau}_2$ is an AUDs scheme, let $\hat{\tau}_2$ be characterized by $(\hat{r}_2, \hat{Q}_2, \hat{p}_2)$, where \hat{Q}_2 is the threshold, \hat{r}_2 and \hat{p}_2 are the pre-threshold and post-threshold prices. In the following we consider the hypothetical situation that

firm 1 uses $\Psi_1(\cdot)$, firm 2 uses $\hat{\tau}_2$, and the buyer uses the tie-breaking rule $\hat{\beta}$. (Of course, firm 2's expected profit would then be π_2 .) Let \hat{q}_2 denote the sales of firm 2 in the outcome under this situation, which is a random variable because its realization depends on the realization of p_1 . We shall prove $\Pr(\hat{q}_2 = k) > 0$ first.

Step 1. It suffices to only consider the case where $\Pr(\hat{q}_2 \geq \hat{Q}_2) > 0$. (In particular, $\hat{Q}_2 \leq k$.) For, if $\Pr(\hat{q}_2 \geq \hat{Q}_2) = 0$, then it is outcome-equivalent to replace $\hat{\tau}_2$ by the AUDs scheme characterized by $(\infty, 0, \hat{r}_2)$ (which is *de facto* a linear pricing with price \hat{r}_2). But now the new \hat{Q}_2 is 0 and $\Pr(\hat{q}_2 \geq 0) = 1$. Therefore, in the following steps, we assume $\Pr(\hat{q}_2 \geq \hat{Q}_2) > 0$ without loss of generality.

Step 2. We have $\Pr(\hat{q}_2 > 0) > 0$. For, otherwise $\pi_2 = 0$, contradicting our previous result.

Step 3. If $\hat{Q}_2 = k$, then Step 1 implies $\Pr(\hat{q}_2 = k) > 0$ as desired. Therefore, in the following steps we suppose $\hat{Q}_2 < k$.

Step 4. Consider the case of $\hat{p}_2 \leq u'(k)$ and $\hat{Q}_2 = 0$. Note that $\hat{\tau}_2$ is *de facto* a linear pricing with price \hat{p}_2 . Then the event $p_1 < \hat{p}_2$ implies the event $\hat{q}_2 = 0$. Since $\hat{p}_2 \leq u'(k)$, the event $p_1 > \hat{p}_2$ implies the event $\hat{q}_2 = k$. If $p_1 = \hat{p}_2$ is a positive probability event, then the buyer's tie-breaking rule $\hat{\beta}$ must favor firm 2 (i.e., $\hat{q}_2 = k$) in this event, for otherwise $\hat{\tau}_2$ cannot be firm 2's best response to $\Psi_1(\cdot)$. Therefore, $\Pr(\hat{q}_2 > 0) = \Pr(\hat{q}_2 = k)$. It together with Step 2 implies $\Pr(\hat{q}_2 = k) > 0$ as desired. Therefore, in the following steps we assume at least one of $\hat{p}_2 > u'(k)$ and $\hat{Q}_2 > 0$ holds.

Step 5. Consider the case of $\hat{p}_2 \leq u'(k)$. From Step 4 we also have $\hat{Q}_2 > 0$. The event $p_1 < \hat{p}_2$ implies the event $\hat{q}_2 = 0 < \hat{Q}_2$. Since $\hat{p}_2 \leq u'(k)$, the event $[\hat{q}_2 \geq \hat{Q}_2$ and $p_1 > \hat{p}_2]$ implies the event $\hat{q}_2 = k$. If $p_1 = \hat{p}_2$ is a positive probability event, then the buyer's tie-breaking rule $\hat{\beta}$ must favor firm 2 (i.e., $\hat{q}_2 = k$ whenever $\hat{q}_2 \geq \hat{Q}_2$) in this event, for otherwise $\hat{\tau}_2$ cannot be firm 2's best response to $\Psi_1(\cdot)$. Therefore,

$$\Pr(\hat{q}_2 \geq \hat{Q}_2) = \Pr(\hat{q}_2 \geq \hat{Q}_2 \text{ and } p_1 \geq \hat{p}_2) \leq \Pr(\hat{q}_2 = k).$$

It together with Step 1 implies $\Pr(\hat{q}_2 = k) > 0$ as desired. Therefore, in the following steps we assume $\hat{p}_2 > u'(k)$.

Step 6. Define $V(p) \equiv \max_{q \geq 0} \{u(q) - pq\}$ as the buyer's surplus when she purchases optimally at per-unit price p . Define \hat{p}_1 as the unique value such that $\hat{p}_1 \leq u'(0)$ and

$$V(\hat{p}_1) = \max_{q \in [\hat{Q}_2, k]} \{u(q) - \hat{p}_2 q\}.$$

Such a \hat{p}_1 is well defined because Step 1 implies $\max_{q \in [\hat{Q}_2, k]} \{u(q) - \hat{p}_2 q\} \geq 0$. (Intuitively, if firm 2 uses the AUDs scheme characterized by $(\infty, \hat{Q}_2, \hat{p}_2)$, then \hat{p}_1 is firm 1's threat price, i.e., firm 2 would sell nothing if firm 1's price is lower than \hat{p}_1 .) Note that $\hat{p}_1 > \hat{p}_2$ if $u'(\hat{Q}_2) < \hat{p}_2$; and $\hat{p}_1 = \hat{p}_2$ otherwise. Now, if $\hat{r}_2 < \hat{p}_1$, then, no matter what firm 1 charges, the buyer would never choose $\hat{q}_2 \geq \hat{Q}_2$, because the buyer prefers to buy on the pre-threshold range more than on the post-threshold range. (Note that $\hat{p}_1 > \hat{r}_2 \geq \hat{p}_2$ implies $u'(\hat{Q}_2) < \hat{p}_2$ and $D(\hat{r}_2) \leq D(\hat{p}_2) < \hat{Q}_2$.) It contradicts Step 1. So suppose $\hat{r}_2 \geq \hat{p}_1$ (and hence \hat{r}_2 is irrelevant).

Now, once firm 1's price p_1 is realized, the buyer's maximum surplus is $\max\{V(p_1), V(\hat{p}_1)\}$. If $p_1 > \hat{p}_1$, the buyer would purchase 0 from firm 1 and $\operatorname{argmax}_{q \in [\hat{Q}_2, k]} \{u(q) - \hat{p}_2 q\} = \max\{D(\hat{p}_2), \hat{Q}_2\}$ from firm 2. (Note that $\max\{D(\hat{p}_2), \hat{Q}_2\} < k$ from Steps 3 and 5.) If $p_1 < \hat{p}_1$, the buyer would purchase 0 from firm 2 and $D(p_1)$ from firm 1. If $p_1 = \hat{p}_1$, the buyer is indifferent between buying only from firm 1 and only from firm 2; but if $p_1 = \hat{p}_1$ is a positive probability event, the buyer would purchase $\max\{D(\hat{p}_2), \hat{Q}_2\}$ from firm 2 in this event, for otherwise \hat{r}_2 cannot be firm 2's best response to $\Psi_1(\cdot)$. Therefore, by using \hat{r}_2 , firm 2's expected profit is

$$\pi_2 = \Pr(p_1 \geq \hat{p}_1) \cdot \hat{p}_2 \cdot \max\{D(\hat{p}_2), \hat{Q}_2\}.$$

Since $\pi_2 > 0$, we know the above three factors are positive.

Step 7. Suppose, by way of contradiction, that $\Pr(\hat{q}_2 = k) = 0$. We show that firm 2 can raise its profit by offering another AUDs scheme, contradicting the hypothesis that \hat{r}_2 is a best response to $\Psi_1(\cdot)$ when the buyer's tie-breaking rule is $\hat{\beta}$. Define p'_2 such that

$$V(\hat{p}_1) = u(k) - p'_2 k.$$

(Intuitively, \hat{p}_1 is firm 1's threat price also when firm 2 uses the AUDs scheme

characterized by (∞, k, p'_2) .) Note that $p'_2 \in (u'(k), \hat{p}_2)$ since $\hat{Q}_2 < k$ and $\hat{p}_1 \geq \hat{p}_2 > u'(k)$ from Steps 3 and 5. By using the AUDs scheme τ'_ε characterized by $(\infty, k, p'_2 - \varepsilon)$ for small $\varepsilon > 0$ and letting $\varepsilon \rightarrow 0$, firm 2's expected profit can be made arbitrarily close to

$$\pi'_2 = \Pr(p_1 \geq \hat{p}_1) \cdot p'_2 k.$$

But $\pi'_2 > \pi_2$ because

$$\begin{aligned} p'_2 k &= u(k) - V(\hat{p}_1) \\ &> u(\max\{D(\hat{p}_2), \hat{Q}_2\}) - V(\hat{p}_1) \\ &= \hat{p}_2 \max\{D(\hat{p}_2), \hat{Q}_2\}, \end{aligned}$$

where the inequality is from $\max\{D(\hat{p}_2), \hat{Q}_2\} < k$ and $u'(k) > 0$. We conclude that $\Pr(\hat{q}_2 = k) > 0$ as desired.

Step 8. The result $\Pr(\hat{q}_2 = k) > 0$ holds for any firm 2's pure-strategy best response to $\Psi_1(\cdot)$. Since in equilibrium firm 2 must randomize over best responses to $\Psi_1(\cdot)$ (i.e., the event that firm 2's offer is a pure-strategy best response to $\Psi_1(\cdot)$ has probability one), we must have $\Pr(q_2 = k) > 0$ in equilibrium. \blacksquare

Proof of Proposition 2. Consider a simultaneous-move \mathcal{T}_1 vs \mathcal{T}_2 game, where $\mathcal{T}_1 = \mathcal{T}^L$ and $\mathcal{T}_2 \in \{\mathcal{T}^A, \mathcal{T}^L\}$. We first construct a SPE for the $\mathcal{T}_2 = \mathcal{T}^L$ case as in [Kreps and Scheinkman \(1983\)](#). Let $\bar{p} \equiv p_1^{L_1 L_2} = p^{cap}$ and let $\underline{p} \equiv p_2^{L_2 L_1}$ (i.e., \underline{p} is the unique value satisfying $\pi(\underline{p}) = \pi^{cap}$ and $\underline{p} < p^{cap}$). Let firm i make offer according to the c.d.f. $\Psi_i(\cdot)$ with support $[\underline{p}, \bar{p}]$, where

$$\Psi_1(p) \equiv \begin{cases} 1 - \frac{p}{\bar{p}} & \text{if } p \in [\underline{p}, \bar{p}) \\ 1 & \text{if } p = \bar{p} \end{cases},$$

$$\Psi_2(p) \equiv \frac{\pi(p) - \pi^{cap}}{pk} \quad \forall p \in [\underline{p}, \bar{p}].$$

It is straightforward to verify that the above strategies together with any buyer's best response (i.e., the buyer's tie-breaking rule does not matter) constitute a SPE when

$\mathcal{T}_2 = \mathcal{T}^L$; the resulting expected profits are as stated in the proposition: $\Pi_1^{\{LL\}} = \pi^{cap}$ and $\Pi_2^{\{LL\}} = \underline{p}k > 0$. We shall show that they also constitute a SPE when $\mathcal{T}_2 = \mathcal{T}^A$, and every SPE results in same profits π^{cap} and $\underline{p}k$.

Consider any (pure-strategy or not) SPE for $\mathcal{T}_2 \in \{\mathcal{T}^A, \mathcal{T}^L\}$. Let π_i denote firm i 's expected profit in the equilibrium. Let p_1 denote the equilibrium offer of firm 1, with c.d.f. denoted by $\Psi_1(\cdot)$. Let \bar{p}_1 and \underline{p}_1 denote the supremum and infimum of the support of $\Psi_1(\cdot)$. Let q_2 denote the equilibrium (random) sales of firm 2.

Step 1. From Lemma A.1 we know $\Pr(q_2 = k) > 0$. The event $p_1 = \bar{p}_1$ implies the event $q_2 = k$. Moreover, if $\Pr(p_1 = \bar{p}_1) = 0$, then $q_2 = k$ holds when the realization of p_1 is close enough to \bar{p}_1 .

If $\Pr(p_1 = \bar{p}_1) > 0$, then firm 1 is willing to offer \bar{p}_1 and hence $\pi_1 = \bar{p}_1 \max\{D(\bar{p}_1) - k, 0\}$. If, on the other hand, $\Pr(p_1 = \bar{p}_1) = 0$, then there exists a positive sequence $\{\varepsilon_n\}$ convergent to 0 such that $\pi_1 = (\bar{p}_1 - \varepsilon_n) \max\{D(\bar{p}_1 - \varepsilon_n) - k, 0\}$, then we again have $\pi_1 = \bar{p}_1 \max\{D(\bar{p}_1) - k, 0\}$. By offering $p_1 \geq \bar{p}_1$, firm 1 earns profit $p \max\{D(p) - k, 0\}$. By offering $p_1 < \bar{p}_1$, firm 1 earns profit at least $p \max\{D(p) - k, 0\}$. For firm 1 to have no profitable deviation, \bar{p}_1 must be maximizing $p \max\{D(p) - k, 0\}$. Therefore, $\bar{p}_1 = p^{cap}$ and $\pi_1 = \pi^{cap}$.

Step 2. Consider $\mathcal{T}_2 = \mathcal{T}^L$. Let p_2 denote the equilibrium offer of firm 2, with c.d.f. denoted by $\Psi_2(\cdot)$. Let \bar{p}_2 and \underline{p}_2 denote the supremum and infimum of the support of $\Psi_2(\cdot)$. Since $\pi_1, \pi_2 > 0$ from Lemma A.1 and $\bar{p}_1 = p^{cap}$ from Step 1, we have $\underline{p}_1, \underline{p}_2 > 0$ and $\bar{p}_2 \leq p^{cap}$, and hence $\underline{p}_1, \underline{p}_2 \leq p^{cap}$. Next, we claim $\underline{p}_1 = \underline{p}_2$. Suppose on the contrary that $\underline{p}_i < \underline{p}_j \leq p^{cap}$. Then $\pi'_j(\underline{p}_i) > k$. Given firm j 's strategy, firm i 's profit is strictly increasing in its offer p_i on $[\underline{p}_i, \underline{p}_i + \varepsilon]$ for small $\varepsilon > 0$, which is a contradiction. It proves that $\underline{p}_1 = \underline{p}_2$. Let $\underline{p} > 0$ denote the common value of \underline{p}_1 and \underline{p}_2 . Next, we claim that $\Pr(p_1 = \underline{p}) = \Pr(p_2 = \underline{p}) = 0$. Suppose on the contrary that $\Pr(p_i = \underline{p}) > 0$. Then firm $j \neq i$ strictly prefers offering a price slightly below \underline{p} to offering a price slightly above \underline{p} . Then \underline{p} must be an isolated element of the support of $\Psi_j(\cdot)$, so that $\Pr(p_j = \underline{p}) > 0$. But then at least one firm would strictly prefer offering a price slightly below \underline{p} to offering \underline{p} , no matter how we let the buyer break the tie when both firms offer \underline{p} , which is a contradiction. It proves that $\Pr(p_1 = \underline{p}) = \Pr(p_2 = \underline{p}) = 0$, and hence both firms do not use pure strategy.

It follows from the above results that $0 < \underline{p} < p^{cap} < u'(k)$, $\pi_1 = \pi(\underline{p})$ and $\pi_2 = \underline{p} \min\{D(\underline{p}), k\} = \underline{p}k$. Recalling $\pi_1 = \pi^{cap}$ from Step 1, we know \underline{p} is the unique value satisfying $\pi(\underline{p}) = \pi^{cap}$ and $\underline{p} < p^{cap}$. In other words, \underline{p} is firm 2's optimal leader's price $p_2^{L_2L_1}$. Finally, $\pi_2 = p_2^{L_2L_1}k$. In other words, π_2 is firm 2's optimal leader's profit $\Pi_2^{L_2L_1}$.

Step 3. Consider $\mathcal{T}_2 = \mathcal{T}^A$. As in the proof of Lemma A.1, let $\hat{\tau}_2$ be a firm 2's pure-strategy best response to $\Psi_1(\cdot)$ when the buyer uses some tie-breaking rule $\hat{\beta}$. Let $\hat{\tau}_2$ be characterized by $(\hat{r}_2, \hat{Q}_2, \hat{p}_2)$. Since $\bar{p}_1 = p^{cap} < u'(k)$ (from Step 1) and $\hat{p}_2 \leq \bar{p}_1$ (for otherwise $\pi_2 = 0$, contradicting Lemma A.1), by using $\hat{\tau}_2$, firm 2 sells k if $p_1 \geq \hat{p}_2$, and sells 0 if $p_1 < \hat{p}_2$. It is exactly like firm 2 offers the linear pricing with unit price \hat{p}_2 (the parameters \hat{r}_2 and \hat{Q}_2 are irrelevant). Therefore, every SPE here has an outcome-equivalent SPE of the simultaneous-move LP vs LP game. In particular, $\pi_2 = \Pi_2^{L_2L_1} = p_2^{L_2L_1}k$ and there is no pure-strategy SPE.

Step 4. It remains to show the existence of SPE for the case of $\mathcal{T}_2 = \mathcal{T}^A$. Indeed, the strategy profile given in the beginning of this proof is also a SPE when $\mathcal{T}_2 = \mathcal{T}^A$. To see this, simply notice that the logic of Step 3 already reveals that, if firm 2 can raise its profit by deviating to some AUDs scheme, it can also do so by deviating to some LP scheme. ■

Proof of Proposition 3. Take any combination of pricing forms $\mathcal{T}_1, \mathcal{T}_2 \in \{\mathcal{T}^A, \mathcal{T}^L\}$ chosen in Stage 0. Let $\hat{\tau}_i \in \mathcal{T}_i$ be a firm i 's strategy in the reduced game that is not "Wait." We claim that offering $\hat{\tau}_i$ is weakly dominated by "Wait" for firm i in the reduced game if either $(\mathcal{T}_1, \mathcal{T}_2) \neq (\mathcal{T}^A, \mathcal{T}^L)$, or $(\mathcal{T}_1, \mathcal{T}_2) = (\mathcal{T}^A, \mathcal{T}^L)$ and $i = 2$.

First, suppose that firm $j \neq i$ in Stage 1 makes an offer $\tau_j \in \mathcal{T}_j$ to which $\hat{\tau}_i$ is a firm i 's best response offer in \mathcal{T}_i . Then clearly firm i is indifferent between choosing "Wait" and offering $\hat{\tau}_i$ in Stage 1.

Second, suppose that firm $j \neq i$ in Stage 1 makes an offer τ_j to which $\hat{\tau}_i$ is not a firm i 's best response offer in \mathcal{T}_i . Then, for firm i , clearly choosing "Wait" is strictly better than offering $\hat{\tau}_i$ in Stage 1.

Third, suppose that firm $j \neq i$ chooses "Wait" in Stage 1. There are several subcases to be considered.

Consider the subcase where $i = 1$ and $(\mathcal{T}_1, \mathcal{T}_2) = (\mathcal{T}^A, \mathcal{T}^A)$. Then, from Proposition 1, firm 1's profit is S^{cap} if it chooses "Wait" in Stage 1. On the other hand, firm 1's profit is at most S^{cap} if firm 1 offers $\hat{\tau}_1$ in Stage 1, for otherwise the total payoff of firm 2 and the buyer would be strictly below $u(k)$ and thus firm 2 would have a profitable deviation in Stage 2.

Consider the subcase where $i = 2$ and $\mathcal{T}_1 = \mathcal{T}^A$. Then, from Proposition 1, firm 2's profit is 0 if it chooses "Wait" in Stage 1. On the other hand, firm 2's profit is nonpositive if firm 2 offers $\hat{\tau}_2$ in Stage 1, for otherwise the total payoff of firm 1 and the buyer would be strictly below $u(q^e)$ and thus firm 1 would have a profitable deviation in Stage 2.

Consider the subcase where $\mathcal{T}_1 = \mathcal{T}^L$ and $i = 1$. Then, from Proposition 2, firm 1's expected profit is π^{cap} if it chooses "Wait" in Stage 1. On the other hand, firm 1's profit is at most π^{cap} if firm 1 offers any price p in Stage 1. It is because firm 2 is able to undercut in Stage 2 whenever $p > 0$.

Consider the subcase where $(\mathcal{T}_1, \mathcal{T}_2) = (\mathcal{T}^L, \mathcal{T}^L)$ and $i = 2$. Then, from Proposition 2, firm 2's expected profit is $\Pi_2^{L_2L_1}$ if it chooses "Wait" in Stage 1. On the other hand, firm 2's profit is at most $\Pi_2^{L_2L_1}$ if firm 2 offers any price in Stage 1. The last claim is simply from the definition of $\Pi_2^{L_2L_1}$.

Consider the subcase where $(\mathcal{T}_1, \mathcal{T}_2) = (\mathcal{T}^L, \mathcal{T}^A)$ and $i = 2$. Then, from Proposition 2, firm 2's expected profit is $\Pi_2^{L_2L_1}$ if it chooses "Wait" in Stage 1. On the other hand, firm 2's profit is at most $\Pi_2^{L_2L_1}$ if firm 2 offers $\hat{\tau}_2$ in Stage 1. To see this, note that although firm 2, the leader, offers some AUDs scheme $\hat{\tau}_2$, what matters is only its post-threshold price \hat{p}_2 . It is because, if firm 1, the follower, offers something below \hat{p}_2 , the buyer buys nothing from firm 2 anyway; if firm 1 offers something above \hat{p}_2 , the buyer buys $\min\{D(\hat{p}_2), k\}$ from firm 2 anyway. The threshold requirement is nonbinding for the buyer, for otherwise firm 1 sells nothing, which is impossible in equilibrium. So firm 2 offering $\hat{\tau}_2$ is exactly like offering a linear pricing with price \hat{p}_2 , from both firms' points of view. ■

Proof of Proposition 4. It follows immediately from the preceding discussion and CTW. ■

References

- Amir, R. and A. Stepanova (2006). Second-mover advantage and price leadership in Bertrand duopoly. *Games and Economic Behavior*, 55(1):1–20.
- Bernheim, B. D. and M. D. Whinston (1986). Common agency. *Econometrica*, 54(4):923–942.
- Bernheim, B. D. and M. D. Whinston (1998). Exclusive dealing. *Journal of Political Economy*, 106(1):64–103.
- Chao, Y. and G. Tan (2017). All-units discounts: Leverage and partial foreclosure in single-product markets. *Canadian Competition Law Review*, 30(1):93–111.
- Chao, Y., G. Tan, and A. C. L. Wong (2018a). All-units discounts as a partial foreclosure device. *The RAND Journal of Economics*, 49(1):155–180.
- Chao, Y., G. Tan, and A. C. L. Wong (2018b). Competition between linear pricing and all-units discounts under capacity asymmetry. Available at SSRN: <http://dx.doi.org/10.2139/ssrn.3217438>.
- Conlon, C. T. and J. H. Mortimer (2017). Efficiency and foreclosure effects of vertical rebates: Empirical evidence. Available at <https://www.bc.edu/content/dam/bc1/schools/mcas/economics/pdf/working-papers/wp863.pdf>.
- Dowrick, S. (1986). von Stackelberg and Cournot duopoly: Choosing roles. *The RAND Journal of Economics*, 17(2):251–260.
- Feess, E. and A. Wohlschlegel (2010). All-unit discounts and the problem of surplus division. *Review of Industrial Organization*, 37:161–178.
- Fu, X. and G. Tan (2019). Abuse of market dominance under China’s Anti-monopoly Law: The case of Tetra Pak. *Review of Industrial Organization*, 54(2):409–434.
- Gal-Or, E. (1985). First mover and second mover advantages. *International Economic Review*, 26(3):649–653.

- Hamilton, J. H. and S. M. Slutsky (1990). Endogenous timing in duopoly games: Stackelberg or Cournot equilibria. *Games and Economic Behavior*, 2(1):29–46.
- Kreps, D. M. and J. A. Scheinkman (1983). Quantity precommitment and Bertrand competition yield Cournot outcomes. *The Bell Journal of Economics*, 14(2):326–337.
- O’Brien, D. and G. Shaffer (1997). Nonlinear supply contracts, exclusive dealing, and equilibrium market foreclosure. *Journal of Economics & Management Strategy*, 6(4):755–785.
- van Damme, E. and S. Hurkens (2004). Endogenous price leadership. *Games and Economic Behavior*, 47(2):404–420.