Online Appendix: Price Discovery in a Matching and Bargaining Market with Aggregate Uncertainty

Artyom Shneyerov[*] Adam C. L. Wong[†]

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A1 Introduction

This online appendix accompanies our paper "Price Discovery in a Matching and Bargaining Market with Aggregate Uncertainty." It extends our model in the paper by assuming traders only observe imperfect signals about their partners' market times and shows that our main results are robust to this extension.

A2 Model

The model here is the same as that in the main text, except that the bargaining between meeting partners is now under the information that we describe below in Subsection A3.2. In particular, in state H, the inflow rate of the buyers is higher than that of the sellers, and the opposite holds in state L, i.e.,

$$\lambda_B^H > \lambda_S^H > 0, \qquad \lambda_S^L > \lambda_B^L > 0. \tag{A1}$$

A3 Equilibrium

As in the main text, we consider steady-state equilibria in which every meeting on the equilibrium path results in trade, which we call *full trade (market) equilibria* for short. The formal definition of full trade equilibria will be given in Subsection A3.4.

[*]CIREQ, CIRANO and Department of Economics, Concordia University, Canada.
[†]Department of Economics, Lingnan University, Hong Kong. Email: adamwong@ln.edu.hk.

A3.1 Steady state

The determination of the steady state is the same as the one in the main text. We only repeat the key equations in the following for completeness.

The steady-state short-side finding rates are

$$\alpha_B^L = \alpha_S^H = \mu, \tag{A2}$$

while the steady-state long-side finding rates are

$$\alpha_B^H = \frac{\delta\mu\lambda_S^H}{(\delta+\mu)\lambda_B^H - \mu\lambda_S^H} < \mu, \quad \alpha_S^L = \frac{\delta\mu\lambda_B^L}{(\delta+\mu)\lambda_S^L - \mu\lambda_B^L} < \mu. \tag{A3}$$

The steady-state cumulative distributions and densities of market times are

$$F_\ell^\omega(t) = 1 - \exp(-(\delta + \alpha_\ell^\omega)t) \qquad \text{for } \omega \in \{L, H\}, \ell \in \{B, S\}, \tag{A4}$$

$$f_\ell^\omega(t) = (\delta + \alpha_\ell^\omega)\exp(-(\delta + \alpha_\ell^\omega)t) \qquad \text{for } \omega \in \{L, H\}, \ell \in \{B, S\}. \tag{A5}$$

In particular, (A3) implies

$$\alpha_B^H \leq \frac{\lambda_S^H}{\lambda_B^H - \lambda_S^H} \cdot \delta, \tag{A6}$$

$$\alpha_S^L \leq \frac{\lambda_B^L}{\lambda_S^L - \lambda_B^L} \cdot \delta. \tag{A7}$$

The probabilities for an unmatched to be ultimately matched are

$$m_\ell^\omega = \frac{\alpha_\ell^\omega}{\delta + \alpha_\ell^\omega} \qquad \text{for } \omega \in \{L, H\}, \ell \in \{B, S\}.$$

Using (A2)–(A3), we have

$$m_B^L = m_S^H = \frac{\mu}{\delta + \mu},$$

$$m_B^H = \frac{\mu\lambda_S^H}{(\delta+\mu)\lambda_B^H}, \quad m_S^L = \frac{\mu\lambda_B^L}{(\delta+\mu)\lambda_S^L}.$$

Note that (A2)–(A3) imply $f_B^L(t)/f_B^H(t)$ and $f_S^H(t)/f_S^L(t)$ are strictly decreasing in t (monotone likelihood ratio properties).

A3.2 Beliefs

Unlike in the main text, here we assume the bargaining between meeting partners is under the following information. Upon each meeting with the buyer's market time denoted as t_B and the seller's market time denoted as t_S, the Nature randomly draws a signal z_B about t_B and a signal z_S about t_S, from c.d.f.'s $G_B(\cdot|t_B)$ and $G_S(\cdot|t_S)$, respectively. Then both traders observe the signals z_B, z_S before their bargaining.

Assumption A1. *For $\ell = B, S$ and any $t \geq 0$, the support of $G_\ell(\cdot|t)$, denoted as $\operatorname{supp} G_\ell(\cdot|t)$, is an interval $[\underline{z}_\ell(t), \bar{z}_\ell(t)]$. $\underline{z}_\ell(t), \bar{z}_\ell(t)$ are continuous and strictly increasing in t. There exists some $\theta \geq 1$ such that, for all $t \geq 0$, $t/\theta \leq \underline{z}_\ell(t) \leq t \leq \bar{z}_\ell(t) \leq \theta t$. $G_\ell(\cdot|t)$ admits a density denoted as $g_\ell(\cdot|t)$. $g_\ell(z|t)$ is continuous on $\{(t, z) \in \mathbb{R}^2_+ : z \in \operatorname{supp} G_\ell(\cdot|t)\}$. Moreover, g_ℓ satisfies the monotone likelihood ratio property: $g_\ell(z'|t')g_\ell(z|t) \geq g_\ell(z'|t)g_\ell(z|t')$ whenever $z' \geq z$ and $t' \geq t$.*

Note that, if $\theta = 1$, the signals z_B, z_S perfectly reveal the market times t_B, t_S and thus the model here reduces to the one in the main text. From the main text we know, when $\theta = 1$, the trading condition holds strictly and hence a full trade equilibrium exists, when the exogenous exit rate δ is small relative to the discount rate r. In the model here the parameter θ is meant to be close enough to 1 (i.e., the signals are not too imprecise) so that a full trade equilibrium exists when δ is small relative to r.

Assumption A1 implies the following, which we state as a lemma without proof.

Lemma A1 (Affiliation)**.** *Take any $t_B, t_S \geq 0$ and $\pi^L, \pi^H \in [0, 1]$ with $\pi^L + \pi^H = 1$. Let $\tilde{\omega} \in \{L, H\}$ be a random variable with $\Pr(\tilde{\omega} = \omega) = \pi^\omega$. Let T_B, T_S be a buyer's and a seller's random market time drawn from c.d.f.'s $F_B^{\tilde{\omega}}, F_S^{\tilde{\omega}}$, and let Z_B, Z_S be random signals about T_B, T_S drawn from c.d.f. $G_B(\cdot|t_B + T_B), G_S(\cdot|t_S + T_S)$. Also declare $L < H$ for the realization of $\tilde{\omega}$. Then, the random variables $\tilde{\omega}, T_B, Z_B, -T_S, -Z_S$ are affiliated (in the sense of Milgrom and Weber (1982)).*

The distributions and densities of the signals, unconditional on the market times, are

$$G_\ell^\omega(z) \equiv \int G_\ell(z|t) f_\ell^\omega(t) dt,$$

$$g_\ell^\omega(z) \equiv \int g_\ell(z|t) f_\ell^\omega(t) dt,$$

for $\omega \in \{L, H\}$, $\ell \in \{B, S\}$, and $t \geq 0$. By Assumption A1, the density $g_\ell^\omega(\cdot)$ is positive and continuous on its support $\mathbb{R}_+$. By Lemma A1, $g_B^H(z)/g_B^L(z)$ and $g_S^L(z)/g_S^H(z)$ are nondecreasing in z on $\mathbb{R}_{++}$.

The distributions and densities of the market times conditional on the signals are

$$F_\ell^\omega(t|z) \equiv \frac{1}{g_\ell^\omega(z)} \left[\int_{t' \le t} g_\ell(z|t') f_\ell^\omega(t') dt' \right],$$

$$f_\ell^\omega(t|z) \equiv \frac{g_\ell(z|t) f_\ell^\omega(t)}{g_\ell^\omega(z)},$$

for $\omega \in \{L, H\}$, $\ell \in \{B, S\}$, and $t, z \ge 0$. The supports of $F_\ell^L(\cdot|z)$ and $F_\ell^H(\cdot|z)$ are the same, both given by the inverse of $\operatorname{supp} G_\ell(\cdot|t)$, which is an interval containing $\{z\}$ and contained in $[z/\theta, \theta z]$. Let $\operatorname{supp} F_\ell(\cdot|z) \equiv [\underline{t}_\ell(z), \bar{t}_\ell(z)]$ denote this common support of $F_\ell^\omega(\cdot|z)$ ($\omega = L, H$). Assumption A1 implies that $\underline{t}_\ell(z), \bar{t}_\ell(z)$, given by the inverses of $\bar{z}_\ell, \underline{z}_\ell$ respectively, are continuous and strictly increasing in z; moreover, $z/\theta \le \underline{t}_\ell(z) \le z \le \bar{t}_\ell(z) \le \theta z$ for all $z \ge 0$.

As in the main text, for any trader we work with his *optimism level*, defined as the ratio of the belief probability of the trader's favorable state (which is L for buyers and H for sellers) to that of his unfavorable state. Upon being born, a buyer's or a seller's initial optimism level is, respectively,

$$\xi_B^0 = \frac{\phi^L \lambda_B^L}{\phi^H \lambda_B^H}, \quad \xi_S^0 = \frac{\phi^H \lambda_S^H}{\phi^L \lambda_S^L}. \tag{A8}$$

As in the main text, we restrict attention to equilibria with passive beliefs.

Assumption A2 (Passive beliefs)**.** *Traders do not update beliefs from off-the-equilibrium-path observations on partners' actions.*

Parallel to the main text, the updating factors for a buyer's or a seller's optimism level, after searching for time t and observing a signal z about a partner's market time, are

$$\xi_B^T(t) \equiv \frac{m_B^L f_B^L(t)}{m_B^H f_B^H(t)}, \quad \xi_B^Z(z) \equiv \frac{g_S^L(z)}{g_S^H(z)}, \tag{A9}$$

$$\xi_S^T(t) \equiv \frac{m_S^H f_S^H(t)}{m_S^L f_S^L(t)}, \quad \xi_S^Z(z) \equiv \frac{g_B^H(z)}{g_B^L(z)}. \tag{A10}$$

Note that, for $\ell = B, S$, ξ_ℓ^T is continuous and strictly decreasing, and ξ_ℓ^Z is continuous and nondecreasing.

Given a current optimism level $\xi \in \overline{\mathbb{R}}_+ \equiv [0, \infty]$, buyers' and sellers' current beliefs about state

ω, denoted as $\pi_B^\omega(\xi), \pi_S^\omega(\xi)$ respectively, are given by

$$\pi_B^L(\xi) = \pi_S^H(\xi) = \frac{\xi}{1+\xi}, \qquad \pi_B^H(\xi) = \pi_S^L(\xi) = \frac{1}{1+\xi}. \tag{A11}$$

A3.3 Bargaining strategies on equilibrium path

In a full trade equilibrium, every meeting on the equilibrium path must be the first meeting of both the buyer and the seller. We claim that, in such an "equilibrium-path meeting," the price offer must depend only on (i) whether the buyer or the seller makes offer, and (ii) the realized signals z_B, z_S about the market times of the two traders. To see this, first note that both traders know this is the first meeting of both of them, and what the buyer (seller) privately knows is his market time t_B (t_S). Say the buyer makes offer. The set of prices acceptable to the seller may depend on the seller's private information t_S and the two traders' public information z_B, z_S. Since the equilibrium is full trade, the buyer must find it optimal to offer the lowest price that is acceptable to the seller of every type $t_S \in \operatorname{supp} F_S(\cdot|z_S)$. This lowest "surely accepted price" depends only on z_B, z_S. Call it $p_B(z_B, z_S)$. Similarly, if the seller makes offer, the price offer must be the highest price acceptable to the buyer of every type $t_B \in \operatorname{supp} F_B(\cdot|z_B)$. Call it $p_S(z_S, z_B)$.

Next, we claim that, in any full trade equilibrium (with passive beliefs), a price offer is never used by the responder to update beliefs, no matter whether the meeting is on or off equilibrium path. To see this, first note that, even if the meeting is off equilibrium path (i.e., not the buyer's and/or the seller's first meeting), in full trade equilibrium both traders still respectively believe this is their partner's first meeting. So the responder expects that the price offer must be $p_B(z_B, z_S)$ if the buyer proposes, or $p_S(z_S, z_B)$ if the seller proposes. If the responder does receive such an offer, by Bayes' rule there would be no updating. Besides, if any other price is proposed, by our assumption of passive beliefs there would be no updating either.

To further characterize the proposing strategy and responding strategy on the equilibrium path, we need to introduce search values. Let $W_B(\xi, t) \in [0, 1]$ denote the buyers' *search value*, i.e., the expected continuation payoff of a currently unmatched buyer with current optimism level ξ and market time t. Similarly let $W_S(\xi, t) \in [0, 1]$ denote the sellers' search value. Assumption A3 below will imply $W_B(\xi, t)$ and $W_S(\xi, t)$ are nondecreasing in ξ and nonincreasing in t.

Consider a buyer who, after searching for time t_B, has just met a seller for the first time, observed signals z_B, z_S, and received a price offer p from the seller. We have seen that the buyer does not use the received offer p to update beliefs. Thus, the buyer's current beliefs are characterized by optimism level $\xi_B^0 \xi_B^T(t_B) \xi_B^Z(z_S)$. If he accepts the offer, he leaves the market with payoff $1 - p$;

should he reject, his continuation payoff would be $W_B(\xi_B^0 \xi_B^T(t_B)\xi_B^Z(z_S), t_B)$. We conclude that a buyer in his first meeting must accept any price offer that is lower than or equal to the cut-off price $\rho_B(t_B, z_S)$ given by

$$\rho_B(t_B, z_S) = 1 - W_B(\xi_B^0 \xi_B^T(t_B)\xi_B^Z(z_S), t_B). \tag{A12}$$

Similarly, a seller in her first meeting must accept any price offer that is higher than or equal to the cut-off price $\rho_S(t_S, z_B)$ given by

$$\rho_S(t_S, z_B) = W_S(\xi_S^0 \xi_S^T(t_S)\xi_S^Z(z_B), t_S). \tag{A13}$$

The monotonicity properties of $W_B, W_S, \xi_B^T, \xi_B^Z, \xi_S^T, \xi_S^Z$ imply that $\rho_B(t_B, z_S)$ is nondecreasing in t_B and nonincreasing in z_S, and $\rho_S(t_S, z_B)$ is nonincreasing in t_S and nondecreasing in z_B.

Now consider proposing strategies. When chosen as a proposer, a buyer in his first meeting must offer $p_B(z_B, z_S)$, which we have seen is the lowest price that is accepted by sellers with any $t_S \in \operatorname{supp} F_S(\cdot|z_S)$. Since $\rho_S(\cdot, z_B)$ is nonincreasing, this lowest surely accepted price is $\sup\{\rho_S(t_S, z_B) : t_S \in \operatorname{supp} F_S(\cdot|z_S)\} = \rho_S(\underline{t}_S(z_S), z_B)$. Using (A13), we have

$$p_B(z_B, z_S) = \rho_S(\underline{t}_S(z_S), z_B) = W_S(\xi_S^0 \xi_S^T(\underline{t}_S(z_S))\xi_S^Z(z_B), \underline{t}_S(z_S)). \tag{A14}$$

Similarly, in her first meeting, a seller must offer

$$p_S(z_S, z_B) = \rho_B(\underline{t}_B(z_B), z_S) = 1 - W_B(\xi_B^0 \xi_B^T(\underline{t}_B(z_B))\xi_B^Z(z_S), \underline{t}_B(z_B)). \tag{A15}$$

The monotonicity properties of $\rho_B, \rho_S, \underline{t}_B, \underline{t}_S$ imply that $p_B(z_B, z_S)$ and $p_S(z_S, z_B)$ are nondecreasing in z_B and nonincreasing in z_S.

A3.4 Bellman equations and full trade equilibrium

Let $V_B(\xi_B, t_B; z_B, z_S) \in [0, 1]$ denote the buyers' *match value*, i.e., the expected continuation payoff of a currently matched buyer who has some market time $t_B \geq 0$, and had some optimism level $\xi_B \in [0, \infty]$ immediately before observing the signals $z_B, z_S \geq 0$ about his own and his current partner's market times. Analogously, let $V_S(\xi_S, t_S; z_S, z_B) \in [0, 1]$ denote the sellers' match value.

Given the match values V_B, V_S, the equilibrium search values W_B, W_S are given by

$$W_B(\xi, t) = \sum_\omega \pi_B^\omega(\xi) m_B^\omega \iiint e^{-rt'_B} V_B(\xi\xi_B^T(t'_B), t + t'_B; z'_B, z'_S) \\ dG_S^\omega(z'_S) dG_B(z'_B | t + t'_B) dF_B^\omega(t'_B), \quad \text{(A16)}$$

$$W_S(\xi, t) = \sum_\omega \pi_S^\omega(\xi) m_S^\omega \iiint e^{-rt'_S} V_S(\xi\xi_S^T(t'_S), t + t'_S; z'_S, z'_B) \\ dG_B^\omega(z'_B) dG_S(z'_S | t + t'_S) dF_S^\omega(t'_S). \quad \text{(A17)}$$

Given the search values W_B, W_S, the equilibrium match values V_B, V_S are given by

$$V_B(\xi_B, t_B; z_B, z_S) = (\beta_B P_B + \beta_S R_B)(\xi_B, t_B; z_B, z_S), \quad \text{(A18)}$$

$$V_S(\xi_S, t_S; z_S, z_B) = (\beta_S P_S + \beta_B R_S)(\xi_S, t_S; z_S, z_B), \quad \text{(A19)}$$

where $R_B(\cdot), R_S(\cdot)$ denote the buyers' and sellers' *responder values*:

$$R_B(\xi_B, t_B; z_B, z_S) = \max\left\{1 - p_S(z_S, z_B), W_B(\xi_B \xi_B^Z(z_S), t_B)\right\}, \quad \text{(A20)}$$

$$R_S(\xi_S, t_S; z_S, z_B) = \max\left\{p_B(z_B, z_S), W_S(\xi_S \xi_S^Z(z_B), t_S)\right\}, \quad \text{(A21)}$$

and $P_B(\cdot), P_S(\cdot)$ denote the buyers' and sellers' *proposer values*:

$$P_B(\xi_B, t_B; z_B, z_S) = \max_p \int \left[(1-p)\mathbf{1}_{\{p \geq \rho_S(t'_S, z_B)\}} + W_B(\xi_B \xi_B^Z(z_S) \xi_B^{\text{rej}}(p, \rho_S; z_B, z_S), t_B)\mathbf{1}_{\{p < \rho_S(t'_S, z_B)\}}\right] \\ d\left(\sum_\omega \pi_B^\omega(\xi_B \xi_B^Z(z_S)) F_S^\omega(t'_S | z_S)\right), \quad \text{(A22)}$$

$$P_S(\xi_S, t_S; z_S, z_B) = \max_p \int \left[p\mathbf{1}_{\{p \leq \rho_B(t'_B, z_S)\}} + W_S(\xi_S \xi_S^Z(z_B) \xi_S^{\text{rej}}(p, \rho_B; z_S, z_B), t_S)\mathbf{1}_{\{p > \rho_B(t'_B, z_S)\}}\right] \\ d\left(\sum_\omega \pi_S^\omega(\xi_S \xi_S^Z(z_B)) F_B^\omega(t'_B | z_B)\right), \quad \text{(A23)}$$

where

$$\xi_B^{\text{rej}}(p,\rho_S;z_B,z_S)\equiv\frac{\int \mathbf{1}_{\{p<\rho_S(t'_S,z_B)\}}dF_S^L(t'_S|z_S)}{\int \mathbf{1}_{\{p<\rho_S(t'_S,z_B)\}}dF_S^H(t'_S|z_S)},\tag{A24}$$

$$\xi_S^{\text{rej}}(p,\rho_B;z_S,z_B)\equiv\frac{\int \mathbf{1}_{\{p>\rho_B(t'_B,z_S)\}}dF_B^H(t'_B|z_B)}{\int \mathbf{1}_{\{p>\rho_B(t'_B,z_S)\}}dF_B^L(t'_B|z_B)}.\tag{A25}$$

The explanations for (A16)–(A21) are analogous to that for their counterparts in the main text. To understand (A22) and (A24), consider a buyer with optimism level and market time (ξ_B, t_B) immediately before the current meeting. Suppose that in the current meeting the realized signals about his and his partner's market times are z_B, z_S and the buyer is chosen as the proposer. The buyer believes this meeting is the seller's first one, and thus updates his optimism level to $\xi_B\xi_B^Z(z_S)$. From his point of view, the seller's market time is a random variable drawn from the distribution $\sum_\omega \pi_B^\omega(\xi_B\xi_B^Z(z_S))F_S^\omega(\cdot|z_S)$. The buyer has to propose some price p without knowing the seller's market time t'_S. If it turns out that $p\geq\rho_S(t'_S,z_B)$, his offer would be accepted by the seller and he would leave the market with payoff $1-p$. If it turns out that $p<\rho_S(t'_S,z_B)$, his offer would be rejected by the seller; from this rejection he would further update his optimism level by multiplying $\xi_B^{\text{rej}}(p,\rho_S;z_B,z_S)$ given in (A24), so that his continuation payoff would be the search value $W_B(\xi_B\xi_B^Z(z_S)\xi_B^{\text{rej}}(p,\rho_S;z_B,z_S),t_B)$. Of course, (A23) and (A25) can be understood similarly.[1]

In any full trade equilibrium, the proposer in any meeting must be willing to make offer according to the proposing strategies $p_B(\cdot), p_S(\cdot)$ specified in the previous subsection, thus the following *trading conditions* must hold: For any $t_B, t_S\geq 0$ and any $z_B\in\operatorname{supp}G_B(\cdot|t_B)$, $z_S\in\operatorname{supp}G_S(\cdot|t_S)$,

$$P_B(\xi_B^0\xi_B^T(t_B),t_B;z_B,z_S)=1-p_B(z_B,z_S)=1-\rho_S(\underline{t}_S(z_S),z_B),\tag{A26}$$

$$P_S(\xi_S^0\xi_S^T(t_S),t_S;z_S,z_B)=p_S(z_S,z_B)=\rho_B(\underline{t}_B(z_B),z_S).\tag{A27}$$

We restrict attention to equilibria with monotone match values, as stated below. While this property can be derived under the model in the main text, under the current model it has to be assumed.

Assumption A3 (Monotone match values)**.** *The match values $V_B(\xi_B,t_B;z_B,z_S)$ and $V_S(\xi_S,t_S;z_S,z_B)$ are jointly continuous in all arguments, nondecreasing in the first and fourth arguments, and nonincreasing in the second and third arguments. (It implies $W_B(\xi,t), W_S(\xi,t)$ given by (A16)–(A17) are*

[1] By the assumption of passive beliefs, $\xi_B^{\text{rej}}(p,\rho_S;z_B,z_S)$ should be defined as 1 for the case that the RHS of (A24) is 0/0. However, this actually does not matter because the RHS of (A22) does not depend on how $\xi_B^{\text{rej}}(p,\rho_S;z_B,z_S)$ is defined for this case. A similar remark applies for $\xi_S^{\text{rej}}(p,\rho_B;z_S,z_B)$.

continuous in (ξ, t), nondecreasing in ξ, and nonincreasing in t, as shown in the proof of Proposition A1.)

Definition A1 (Full trade equilibrium)**.** A *full trade (market) equilibrium* is a tuple $(W_\ell, V_\ell, R_\ell, P_\ell, p_\ell, \rho_\ell)_{\ell=B,S}$ satisfying (A12)–(A27) and Assumption A3.

A3.5 No uncertainty benchmark

There is no difference between the no uncertainty benchmark (where $\phi^\omega = 1$) for the current model and that for the model in the main text. We only repeat the key equations in the following for completeness.

The Bellman equations become

$$\overline{V}_B^\omega = \beta_B \max\left\{1 - \overline{W}_S^\omega, \overline{W}_B^\omega\right\} + \beta_S \overline{W}_B^\omega, \tag{A28}$$

$$\overline{V}_S^\omega = \beta_S \max\left\{1 - \overline{W}_B^\omega, \overline{W}_S^\omega\right\} + \beta_B \overline{W}_S^\omega, \tag{A29}$$

$$\overline{W}_B^\omega = m_B^\omega \int e^{-rt'_B} \overline{V}_B^\omega dF_B^\omega(t'_B) = \frac{\alpha_B^\omega}{r + \delta + \alpha_B^\omega} \overline{V}_B^\omega, \tag{A30}$$

$$\overline{W}_S^\omega = m_S^\omega \int e^{-rt'_S} \overline{V}_S^\omega dF_S^\omega(t'_S) = \frac{\alpha_S^\omega}{r + \delta + \alpha_S^\omega} \overline{V}_S^\omega, \tag{A31}$$

where $\overline{W}_B^\omega, \overline{W}_S^\omega$ and $\overline{V}_B^\omega, \overline{V}_S^\omega$ denote the buyers' and sellers' search values and match values in state ω. From the main text we know they can be rewritten as

$$\overline{V}_B^\omega = \beta_B \left(1 - \overline{W}_S^\omega\right) + \beta_S \overline{W}_B^\omega, \tag{A32}$$

$$\overline{V}_S^\omega = \beta_S \left(1 - \overline{W}_B^\omega\right) + \beta_B \overline{W}_S^\omega, \tag{A33}$$

$$\overline{W}_B^\omega = \frac{\beta_B \alpha_B^\omega}{r + \delta + \beta_B \alpha_B^\omega + \beta_S \alpha_S^\omega}, \tag{A34}$$

$$\overline{W}_S^\omega = \frac{\beta_S \alpha_S^\omega}{r + \delta + \beta_B \alpha_B^\omega + \beta_S \alpha_S^\omega}. \tag{A35}$$

The certainty benchmark prices are $\overline{p}_B^\omega = \overline{W}_S^\omega$ (when the buyer proposes) and $\overline{p}_S^\omega = 1 - \overline{W}_B^\omega$ (when the seller proposes).

Of course, Proposition 1 in the main text also applies here. In particular, if the true state $\omega \in \{L, H\}$ is commonly known, the discrepancy between equilibrium transaction prices (i.e.,

$\overline{p}_B^\omega = \overline{W}_S^\omega$ and $\overline{p}_S^\omega = 1 - \overline{W}_B^\omega$) and the Walrasian price (i.e., 1 for state H and 0 for state L) is of order $r + \delta$.

A4 Uniqueness and basic equilibrium properties

From now on we tackle the aggregate uncertainty case, i.e., $\phi^\omega \in (0, 1)$. As in the main text, we consider *full trade equilibrium candidates*, which in the current model are defined as in Definition A1 except that the trading conditions (A26) and (A27) are neglected. Of course, all the properties of the full trade equilibrium candidates (except existence) are automatically inherited by the full trade equilibria.

We first prove the existence and uniqueness of the full trade equilibrium candidate. As in the main text, it is proved by applying the Contraction Mapping Theorem, but here the complete argument is much more complex.

Proposition A1. *Under any friction profile $(r, \delta) \in \mathbb{R}_+ \times \mathbb{R}_{++}$, there exists a unique full trade equilibrium candidate.*

Proof. Let $\mathcal{B}$ denote the set of $V \equiv (V_B, V_S)$ such that each V_ℓ ($\ell = B, S$) is a function from[2]

$$D_\ell \equiv \{(\xi_\ell, t_\ell; z_\ell, z_{\ell'}) \in \overline{\mathbb{R}}_+ \times \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+ : z_\ell \in \operatorname{supp} G_\ell(\cdot|t_\ell)\}$$

to $[0, 1]$. Let $\mathcal{B}'$ denote the set of $V \in \mathcal{B}$ such that each V_ℓ is jointly continuous in the four arguments, nondecreasing in the first and fourth arguments, and nonincreasing in the second and third arguments. Let $\mathcal{B}''$ denote the set of $V \in \mathcal{B}'$ such that each V_ℓ is bounded from below by $\beta_\ell(r + \delta)/(r + \delta + \mu)$.[3]

If $V_B(\cdot), V_S(\cdot)$ are the match values of any full trade equilibrium candidate, then, by Assumption A3, $V \equiv (V_B, V_S) \in \mathcal{B}'$. Define a mapping $\mathcal{T} : \mathcal{B}' \to \mathcal{B}$ as follows. For any $V \in \mathcal{B}'$, let $W \equiv (W_B, W_S)$, where $W_B, W_S : \overline{\mathbb{R}}_+ \times \mathbb{R}_+ \to [0, 1]$ are given by (A16) and (A17); then let $\rho \equiv (\rho_B, \rho_S)$ and $p \equiv (p_B, p_S)$, where $\rho_B, \rho_S, p_B, p_S : \mathbb{R}_+ \times \mathbb{R}_+ \to [0, 1]$ are given by (A12)–(A15); then let $R \equiv (R_B, R_S)$ and $P \equiv (P_B, P_S)$, where $R_B, P_B : D_B \to [0, 1]$ and $R_S, P_S : D_S \to [0, 1]$ are given by (A20)–(A25); then let $\mathcal{T}(V) \equiv (\mathcal{T}_B(V), \mathcal{T}_S(V))$, where $\mathcal{T}_B(V) : D_B \to [0, 1]$ and $\mathcal{T}_S(V) : D_S \to [0, 1]$ are given by the RHS of (A18) and (A19).

[2] As in the main text, $\overline{\mathbb{R}}_+$ denotes $[0, \infty]$.

[3] The purpose of imposing this lower bound is to guarantee that, in the argument below, $\rho_B(\cdot, z), \rho_S(\cdot, z)$ are *strictly* monotone, which in turn requires $W_B(\cdot, t), W_S(\cdot, t)$ are *strictly* monotone.

Clearly, the set of full trade equilibrium candidates can be identified with the set of fixed points of $\mathcal{T} : \mathcal{B}' \to \mathcal{B}$. If V is a fixed point of $\mathcal{T}|_{\mathcal{B}''}$, clearly V is also a fixed point of $\mathcal{T}$. Conversely, we claim that, if V is a fixed point of $\mathcal{T}$, then $V \in \mathcal{B}''$ so that V is also a fixed point of $\mathcal{T}|_{\mathcal{B}''}$. Let V be a fixed point of $\mathcal{T}$. Then (A16) and $0 \le V_B(\cdot) \le 1$ imply that, for any $\xi \in \overline{\mathbb{R}}_+$, $t \in \mathbb{R}_+$,

$$0 \le W_B(\xi, t) \le \sum_{\omega} \pi_B^{\omega}(\xi) m_B^{\omega} \int e^{-rt'_B} dF_B^{\omega}(t'_B) \le \frac{\mu}{r + \delta + \mu}.$$

So $0 \le W_B(\cdot) \le \mu/(r+\delta+\mu)$. Similarly, (A17) and $0 \le V_S(\cdot) \le 1$ imply $0 \le W_S(\cdot) \le \mu/(r+\delta+\mu)$. Then (A12) and (A13) imply

$$\rho_B(\cdot) \ge 1 - \frac{\mu}{r + \delta + \mu}, \quad \rho_S(\cdot) \le \frac{\mu}{r + \delta + \mu}.$$

Then (A22) and (A23) imply (since the choice p can be taken as $\frac{\mu}{r+\delta+\mu}$ and $1 - \frac{\mu}{r+\delta+\mu}$ respectively)

$$P_B(\cdot), P_S(\cdot) \ge \frac{r + \delta}{r + \delta + \mu}.$$

Also, (A14), (A15), (A20), (A21), and $W_B(\cdot), W_S(\cdot) \ge 0$ imply $R_B(\cdot), R_S(\cdot) \ge 0$. Then (A18) and (A19) imply

$$V_B(\cdot) = \mathcal{T}_B(V)(\cdot) \ge \frac{\beta_B(r + \delta)}{r + \delta + \mu}, \quad V_S(\cdot) = \mathcal{T}_S(V)(\cdot) \ge \frac{\beta_S(r + \delta)}{r + \delta + \mu},$$

as desired. Therefore, the set of full trade equilibrium candidates can be identified with the set of fixed points of $\mathcal{T}|_{\mathcal{B}''}$. In the following we show that $\mathcal{T}|_{\mathcal{B}''}$ has a unique fixed point by applying the Contraction Mapping Theorem.

Claim. $\mathcal{T}|_{\mathcal{B}''}$ is a self-map, i.e. $\mathcal{T}(\mathcal{B}'') \subseteq \mathcal{B}''$.

Proof. Fix any $V \in \mathcal{B}''$. The above arguments have already revealed that $0 < \beta_\ell(r+\delta)/(r+\delta+\mu) \le \mathcal{T}_\ell(V)(\cdot) \le 1$. It remains to show the continuity and monotonicity properties of $\mathcal{T}_\ell(V)$. Consider W_B given by V and (A16). We claim that $W_B(\xi, t)$ is continuous in (ξ, t), nonincreasing in t, and strictly increasing in ξ. The continuity follows from the continuity of V and Assumption A1. Let t increase. It has a direct effect that makes the value of V_B in the integrand in (A16) weakly smaller. It also has an indirect effect via an increase in z'_B in the sense of stochastic dominance, which in turn makes the value of V_B weakly smaller in expectation. Thus, $W_B(\xi, t)$ is nonincreasing in t. Now let ξ increase. The increase in ξ makes the triple integral in (A16) weakly larger, for both $\omega = L$ and $\omega = H$. The integral is weakly larger when evaluated at $\omega = L$ than when evaluated at $\omega = H$, because t'_B, z'_B are smaller and z'_S larger when $\omega = L$ in the sense of stochastic dominance, which in turn makes the value of V_B and $e^{-rt'_B}$ weakly larger in expectation. Also note that the integral, even when evaluated at $\omega = H$, is positive because $V \in \mathcal{B}''$ has a positive lower bound. These together with $m_B^L > m_S^H$ imply that the product of m_B^ω and the integral is strictly larger when evaluated at $\omega = L$ than when evaluated at $\omega = H$. Recalling that $\pi_B^L(\xi)$ is strictly increasing in ξ, we obtain that $W_B(\xi, t)$ is strictly increasing in ξ. Similarly, $W_S(\xi, t)$ given by (A17) is continuous in (ξ, t), nonincreasing in t, and strictly increasing in ξ.

Then it follows from (A14), (A15), (A20), and (A21) that R_B, R_S are continuous, nondecreasing in the first and fourth arguments, and nonincreasing in the second and third arguments. It remains to show that P_B, P_S given by (A22) and (A23) have the same continuity and monotonicity properties.

The continuity and monotonicity properties of P_B, P_S rely on the following observations. First, the continuity and monotonicity properties of W_B, ξ_B^T, ξ_B^Z imply that $\rho_B(t_B, z_S)$ given by (A12) is continuous, strictly increasing in t_B, and nonincreasing z_S; similarly, $\rho_S(t_S, z_B)$ given by (A13) is continuous, strictly decreasing in t_S, and nondecreasing in z_B. So, for any $p \in \{\rho_B(t_B, z_S) : t_B \in \operatorname{supp} F_B(\cdot|z_B)\}$, there is a unique $\rho_B^{-1}(p, z_S) \in \operatorname{supp} F_B(\cdot|z_B)$ such that $\{t_B : p \le \rho_B(t_B, z_S)\} = [\rho_B^{-1}(p, z_S), \infty)$, and this $\rho_B^{-1}(p, z_S)$ is strictly increasing in p and nondecreasing in z_S. Similarly, for any $p \in \{\rho_S(t_S, z_B) : t_S \in \operatorname{supp} F_S(\cdot|z_S)\}$, there is a unique $\rho_S^{-1}(p, z_B) \in \operatorname{supp} F_S(\cdot|z_S)$ such that $\{t_S : p \ge \rho_S(t_S, z_B)\} = [\rho_S^{-1}(p, z_B), \infty)$, and this $\rho_S^{-1}(p, z_B)$ is strictly decreasing p and nondecreasing in z_B. It follows that the acceptance probability in (A22)

$$\int \mathbf{1}_{\{p \ge \rho_S(t'_S, z_B)\}} d\left(\sum_\omega \pi_B^\omega(\xi_B \xi_B^Z(z_S)) F_S^\omega(t'_S|z_S)\right)$$

is nondecreasing in ξ_B, strictly increasing in p, nonincreasing in ρ_S, nonincreasing in z_B, and nondecreasing in z_S, on the set of $(\xi_B, p, \rho_S, z_B, z_S)$ such that the above probability is strictly between 0 and 1; similarly, the acceptance probability in (A23) is nondecreasing ξ_S, strictly decreasing in p, nondecreasing in ρ_B, nonincreasing in z_S, and nondecreasing in z_B, on the set of $(\xi_S, p, \rho_B, z_S, z_B)$ such that the probability is strictly between 0 and 1. Moreover, considering random variables $\tilde{\omega}, T_B, T_S, Z_B, Z_S$ as in Lemma A1 with $\Pr(\tilde{\omega} = L) = \Pr(\tilde{\omega} = H) = 1/2$, we have, whenever the RHS of (A24) is not 0/0 (i.e., whenever $p < \rho_S(\underline{t}_S(z_S), z_B)$),

$$\xi_B^Z(z_S)\xi_B^{\text{rej}}(p, \rho_S; z_B, z_S), t_B) = \frac{\Pr\left(\tilde{\omega} = L | Z_S = z_S, T_S < \rho_S^{-1}(p, z_B)\right)}{\Pr\left(\tilde{\omega} = H | Z_S = z_S, T_S < \rho_S^{-1}(p, z_B)\right)},$$

which by Lemma A1 is nonincreasing in p and nondecreasing in ρ_S, z_B, z_S, and by Assumption A1 is continuous. Similarly, whenever the RHS of (A25) is not 0/0 (i.e., whenever $p > \rho_B(\underline{t}_B(z_B), z_S)$), we have

$$\xi_S^Z(z_B)\xi_S^{\text{rej}}(p, \rho_B; z_S, z_B), t_S) = \frac{\Pr\left(\tilde{\omega} = H | Z_B = z_B, T_B < \rho_B^{-1}(p, z_S)\right)}{\Pr\left(\tilde{\omega} = L | Z_B = z_B, T_B < \rho_B^{-1}(p, z_S)\right)},$$

which is nondecreasing in p, z_B, z_S, nonincreasing in ρ_B, and continuous.

Now consider $P_B(\xi_B, t_B; z_B, z_S)$ given by (A22) and let p^* be an optimal price offer for the maximization problem in $P_B(\xi_B, t_B; z_B, z_S)$. Let $\hat{\xi}_B \geq \xi_B$, $\hat{t}_B \leq t_B$, $\hat{z}_B \leq z_B$, and $\hat{z}_S \geq z_S$. If, in the maximization problem in $P_B(\xi_B, t_B; z_B, z_S)$, the probability that p^* is accepted is 0, then $P_B(\xi_B, t_B; z_B, z_S) = W_B(\xi_B \xi_B^Z(z_S), t_B)$; then, by picking an suboptimal $\hat{p}$ that is accepted with probability 0 for the maximization problem in $P_B(\hat{\xi}_B, \hat{t}_B; \hat{z}_B, \hat{z}_S)$, we see $P_B(\hat{\xi}_B, \hat{t}_B; \hat{z}_B, \hat{z}_S) \geq W_B(\hat{\xi}_B \xi_B^Z(\hat{z}_S), \hat{t}_B) \geq W_B(\xi_B \xi_B^Z(z_S), t_B) = P_B(\xi_B, t_B; z_B, z_S)$, as desired. So suppose that p^* is accepted with positive probability in $P_B(\xi_B, t_B; z_B, z_S)$. Then it must be the case that $1 - p^* \geq W_B(\xi_B \xi_B^Z(z_S) \xi_B^{\text{rej}}(p^*, \rho_S; z_B, z_S), t_B)$ (that is, the buyer prefers his offer p^* to be accepted), for otherwise offering slightly below p^* would increase the acceptance payoff $1 - p$, without decreasing the rejection probability and the rejection payoff $W_B(\xi_B \xi_B^Z(z_S) \xi_B^{\text{rej}}(p, \rho_S; z_B, z_S), t_B)$, contradicting that p^* is optimal. To see that $P_B(\cdot, t_B; z_B, z_S)$ is nondecreasing, pick the suboptimal $\hat{p}$ for the maximization problem in $P_B(\hat{\xi}_B, t_B; z_B, z_S)$ such that

$$\int \mathbf{1}_{\{\hat{p} \geq \rho_S(t'_S, z_B)\}} d\left(\sum_\omega \pi_B^\omega(\hat{\xi}_B \xi_B^Z(z_S)) F_S^\omega(t'_S | z_S)\right) = \int \mathbf{1}_{\{p \geq \rho_S(t'_S, z_B)\}} d\left(\sum_\omega \pi_B^\omega(\xi_B \xi_B^Z(z_S)) F_S^\omega(t'_S | z_S)\right),$$

then $\hat{p} \leq p^*$. We see $P_B(\hat{\xi}_B, t_B; z_B, z_S) \geq P_B(\xi_B, t_B; z_B, z_S)$ as desired. To see that $P_B(\xi_B, \cdot; z_B, z_S)$ is nonincreasing, simply recall that $W_B(\xi, \cdot)$ is nonincreasing. To see that $P_B(\xi_B, t_B; \cdot, z_S)$ is nonincreasing, recall that $\hat{z}_B \leq z_B$ implies $\rho_S(\cdot, \hat{z}_B) \leq \rho_S(\cdot, z_B)$ and then pick the suboptimal $\hat{p} = \rho_S(\rho_S^{-1}(p^*, z_B), \hat{z}_B)$ for the maximization problem in $P_B(\xi_B, t_B; \hat{z}_B, z_S)$; then $\{t_S : p^* \geq \rho_S(t_S, z_B)\} = \{t_S : \hat{p} \geq \rho_S(t_S, \hat{z}_B)\}$ and $\hat{p} \geq p^*$. We see that $P_B(\xi_B, t_B; \hat{z}_B, z_S) \geq P_B(\xi_B, t_B; z_B, z_S)$ as desired. To see that $P_B(\xi_B, t_B; z_B, \cdot)$ is nondecreasing, pick the suboptimal $\hat{p}$ for the maximization problem in $P_B(\xi_B, t_B; z_B, \hat{z}_S)$ such that

$$\int \mathbf{1}_{\{\hat{p} \geq \rho_S(t'_S, z_B)\}} d\left(\sum_\omega \pi_B^\omega(\xi_B \xi_B^Z(\hat{z}_S)) F_S^\omega(t'_S | \hat{z}_S)\right) = \int \mathbf{1}_{\{p \geq \rho_S(t'_S, z_B)\}} d\left(\sum_\omega \pi_B^\omega(\xi_B \xi_B^Z(z_S)) F_S^\omega(t'_S | z_S)\right),$$

then $\hat{p} \leq p^*$. We see $P_B(\xi_B, t_B; z_B, \hat{z}_S) \geq P_B(\xi_B, t_B; z_B, z_S)$ as desired. Moreover, $P_B(\cdot)$ is continuous by the Maximum Theorem. We conclude that $P_B(\cdot)$ is continuous, nondecreasing in the first and fourth arguments, and nonincreasing in the second and third arguments. Similarly, $P_S(\cdot)$ has the same properties. Finally, $\mathcal{T}_B(V), \mathcal{T}_S(V)$ given by (A18) and (A19) have the same continuity and monotonicity properties. ■

Claim. $\mathcal{T}|_{\mathcal{B}''}$ is a contraction on the nonempty complete metric space $(\mathcal{B}'', d_\infty)$, where d_∞ denotes the supremum metric (or uniform metric).

Proof. Consider any $V, \hat{V} \in \mathcal{B}''$ and let $\hat{d} \equiv d_\infty(V, \hat{V})$. Then $W_B, \hat{W}_B$ given by (A16) satisfy, for any $(\xi, t) \in \overline{\mathbb{R}}_+ \times \mathbb{R}_+$,

$$\begin{aligned}
(W_B - \hat{W}_B)(\xi, t) &\leq \sum_\omega \pi_B^\omega(\xi) m_B^\omega \int e^{-rt'_B} \hat{d} dF_B^\omega(t'_B) \\
&= \sum_\omega \pi_B^\omega(\xi) \frac{\alpha_B^\omega}{r + \delta + \alpha_B^\omega} \hat{d} \\
&\leq \max\left\{ \frac{\alpha_B^L}{r + \delta + \alpha_B^L}, \frac{\alpha_B^H}{r + \delta + \alpha_B^H} \right\} \hat{d} \\
&= \frac{\mu}{r + \delta + \mu} \cdot \hat{d}.
\end{aligned}$$

Let $k \equiv \frac{\mu}{r+\delta+\mu} < 1$. Then $d_\infty(W_B, \hat{W}_B) \leq k\hat{d}$ and similarly $d_\infty(W_S, \hat{W}_S) \leq k\hat{d}$. From (A12)–(A15), it is clear that

$$d_\infty(\rho_B, \hat{\rho}_B), d_\infty(\rho_S, \hat{\rho}_S), d_\infty(p_B, \hat{p}_B), d_\infty(p_S, \hat{p}_S) \leq k\hat{d}.$$

Since for any $x_1, x_2, x_3, x_4 \in \mathbb{R}$,[4]

$$|\max\{x_1, x_2\} - \max\{x_3, x_4\}| \leq \max\{|x_1 - x_3|, |x_2 - x_4|\},$$

$R_B, \hat{R}_B, R_S, \hat{R}_S$ given by (A20) and (A21) satisfy

$$d_\infty(R_B, \hat{R}_B) \leq \max\left\{ d_\infty(p_S, \hat{p}_S), d_\infty(W_B, \hat{W}_B) \right\} \leq k\hat{d},$$

$$d_\infty(R_S, \hat{R}_S) \leq \max\left\{ d_\infty(p_B, \hat{p}_B), d_\infty(W_S, \hat{W}_S) \right\} \leq k\hat{d}.$$

Now we show $d_\infty(P_B, \hat{P}_B) \leq k\hat{d}$, where $P_B, \hat{P}_B$ are given by (A22). Fix any $(\xi_B, t_B, z_B, z_S) \in$

[4] This follows from the following chain of inequalities, assuming without loss of generality $x_1 = \max\{x_1, x_2, x_3, x_4\}$:

$|\max\{x_1, x_2\} - \max\{x_3, x_4\}| = |x_1 - \max\{x_3, x_4\}| = x_1 - \max\{x_3, x_4\} \leq x_1 - x_3 \leq \max\{|x_1 - x_3|, |x_2 - x_4|\}$.

D and without loss of generality assume $P_B(\xi_B, t_B; z_B, z_S) \geq \hat{P}_B(\xi_B, t_B; z_B, z_S)$. Let p^* be an optimal price offer for the maximization problem in $P_B(\xi_B, t_B, z_B, z_S)$. If, in the maximization problem in $P_B(\xi_B, t_B; z_B, z_S)$, the probability that p^* is accepted is 0, then $P_B(\xi_B, t_B; z_B, z_S) = W_B(\xi_B \xi_B^Z(z_S), t_B)$; then, by picking an suboptimal $\hat{p}$ that is accepted with probability 0 for the maximization problem in $\hat{P}_B(\xi_B, t_B; z_B, z_S)$, we see $\hat{P}_B(\xi_B, t_B; z_B, z_S) \geq \hat{W}_B(\xi_B \xi_B^Z(z_S), t_B)$ and hence

$$(P_B - \hat{P}_B)(\xi_B, t_B; z_B, z_S) \leq (W_B - \hat{W}_B)(\xi_B \xi_B^Z(z_S), t_B) \leq d_\infty(W_B, \hat{W}_B) \leq k\hat{d},$$

as desired. So suppose that p^* is accepted with positive probability in $P_B(\xi_B, t_B; z_B, z_S)$, i.e., $p^* > \rho_S(\bar{t}_S(z_S), z_B)$. We have seen that $\rho_S(\cdot, z_B), \hat{\rho}_S(\cdot, z_B)$ are continuous and strictly decreasing in the proof of the previous claim. Since p^* is optimal in $P_B(\xi_B, t_B; z_B, z_S)$, we know $p^* \leq \rho_S(\underline{t}_S(z_S), z_B)$. These imply that there is a unique $t_S^* \equiv \rho_S^{-1}(p^*, z_B) \geq z_S$ such that $\{t_S : p^* \geq \rho_S(t_S, z_B)\} = [t_S^*, \infty)$. For the maximization problem in $\hat{P}_B(\xi_B, t_B; z_B, z_S)$, pick the suboptimal price offer $\hat{p} \equiv \hat{\rho}_S(t_S^*, z_B)$, so that $\{t_S : \hat{p} \geq \hat{\rho}_S(t_S, z_B)\} = [t_S^*, \infty)$. So,

$$P_B(\xi_B, t_B; z_B, z_S) = \int \left[(1 - \rho_S(t_S^*, z_B))\mathbf{1}_{t'_S \geq t_S^*} + W_B(\xi_B \xi_B^{\text{rej}}(p^*, \rho_S; z_B, z_S), t_B)\mathbf{1}_{t'_S < t_S^*}\right] d\left(\sum_\omega \pi_B^\omega(\xi_B \xi_B^Z(z_S)) F_S^\omega(t'_S | z_S)\right),$$

$$\hat{P}_B(\xi_B, t_B; z_B, z_S) \geq \int \left[(1 - \hat{\rho}_S(t_S^*, z_B))\mathbf{1}_{t'_S \geq t_S^*} + \hat{W}_B(\xi_B \xi_B^{\text{rej}}(\hat{p}, \hat{\rho}_S; z_B, z_S), t_B)\mathbf{1}_{t'_S < t_S^*}\right] d\left(\sum_\omega \pi_B^\omega(\xi_B \xi_B^Z(z_S)) F_S^\omega(t'_S | z_S)\right).$$

By our construction, the probability that $\hat{p}$ is accepted in the problem in $\hat{P}_B(\xi_B, t_B; z_B, z_S)$ is equal to the probability that p^* is accepted in the problem in $P_B(\xi_B, t_B; z_B, z_S)$; moreover, $\xi_B^{\text{rej}}(\hat{p}, \hat{\rho}_S; z_B, z_S), t_B) = \xi_B^{\text{rej}}(p^*, \rho_S; z_B, z_S), t_B)$. Hence,

$$\begin{aligned}(P_B - \hat{P}_B)(\xi_B, t_B; z_B, z_S) &\leq \max\{(\hat{\rho}_S - \rho_S)(t_S^*, z_B), (W_B - \hat{W}_B)(\xi_B \xi_B^{\text{rej}}(p^*, \rho_S; z_B, z_S), t_B)\} \\ &\leq \max\{d_\infty(\rho_S, \hat{\rho}_S), d_\infty(W_B, \hat{W}_B)\} \leq k\hat{d},\end{aligned}$$

as desired. We conclude that $d_\infty(P_B, \hat{P}_B) \leq k\hat{d}$; similarly, one can prove $d_\infty(P_S, \hat{P}_S) \leq k\hat{d}$. Now,

$\mathcal{T}_B(V), \mathcal{T}_B(\hat{V}), \mathcal{T}_S(V), \mathcal{T}_S(\hat{V})$ given by (A18) and (A19) satisfy $d_\infty(\mathcal{T}_B(V), \mathcal{T}_B(\hat{V})), d_\infty(\mathcal{T}_S(V), \mathcal{T}_S(\hat{V}) \leq k\hat{d}$. We conclude that $\mathcal{T}|_{\mathcal{B}''} : \mathcal{B}'' \to \mathcal{B}''$ is a contraction with contraction coefficient $\frac{\mu}{r+\delta+\mu} < 1$. ■

Finally, by the Contraction Mapping Theorem, $\mathcal{T}|_{\mathcal{B}''} : \mathcal{B}'' \to \mathcal{B}''$ has a unique fixed point, as desired. ■

As in the main text, the Contraction Mapping Theorem also allows us to establish some basic properties of the full trade equilibrium candidate.

Proposition A2 (Basic equilibrium properties)**.** *In any full trade equilibrium, $W_B(\xi, t), W_S(\xi, t)$ are continuous in (ξ, t), strictly increasing in ξ, and nonincreasing in t; moreover,*

$$\overline{V}_B^H \leq V_B(\cdot) \leq \overline{V}_B^L, \quad \overline{V}_S^L \leq V_S(\cdot) \leq \overline{V}_S^H, \tag{A36}$$

$$\overline{W}_B^H \leq W_B(\cdot) \leq \overline{W}_B^L, \quad \overline{W}_S^L \leq W_S(\cdot) \leq \overline{W}_S^H. \tag{A37}$$

(A37) in particular implies $\overline{p}_B^L \leq p_B(\cdot) \leq \overline{p}_B^H$ and $\overline{p}_S^L \leq p_S(\cdot) \leq \overline{p}_S^H$; that is, relative to the no uncertainty benchmarks, the equilibrium prices include a discount in state H and a premium in state L.

Proof. Here we follow the notations and results established in the proof of Proposition A1. In that proof, we have seen that the self-map $\mathcal{T}|_{\mathcal{B}''}$ is a contraction on the nonempty complete metric space $(\mathcal{B}'', d_\infty)$, where d_∞ denotes the supremum metric, and the unique full trade equilibrium candidate can be identified with the unique fixed point of $\mathcal{T}$. The continuity and monotonicity properties of W_B, W_S have already been revealed. Let $\mathcal{B}'''$ denote the set of $V \equiv (V_B, V_S) \in \mathcal{B}''$ satisfying (A36). In the following we show that, for any $V \in \mathcal{B}'''$, we have $\mathcal{T}(V) \in \mathcal{B}'''$. So fix any $V \in \mathcal{B}'''$. Since $\alpha_B^H < \alpha_B^L$ (and hence $F_B^H(\cdot)$ first-order stochastically dominates $F_B^L(\cdot)$) and $\alpha_S^L < \alpha_S^H$ (and hence $F_S^L(\cdot)$ first-order stochastically dominates $F_S^H(\cdot)$), (A16), (A17), and (A36) imply

$$W_B(\cdot) \geq m_B^H \int e^{-rt'_B} \overline{V}_B^H dF_B^H(t'_B) = \overline{W}_B^H,$$

$$W_S(\cdot) \leq m_S^H \int e^{-rt'_S} \overline{V}_S^H dF_S^H(t'_S) = \overline{W}_S^H.$$

Then (A12)–(A15) imply $\rho_B(\cdot), p_S(\cdot) \leq 1 - \overline{W}_B^H$ and $\rho_S(\cdot), p_B(\cdot) \leq \overline{W}_S^H$. Then (A20) and (A21) imply $R_B(\cdot) \geq \overline{W}_B^H$ and $R_S(\cdot) \leq \overline{W}_S^H$. Then (A22) implies $P_B(\cdot) \geq 1 - \overline{W}_S^H$ because the buyer can offer $p = \overline{W}_S^H$. (A23) implies $P_S(\cdot) \le \max\{1 - \overline{W}_B^H, \overline{W}_S^H\} = 1 - \overline{W}_B^H$. Then (A18) implies

$$\mathcal{T}_B(V)(\cdot) \ge \beta_B \left(1 - \overline{W}_S^H\right) + \beta_S \overline{W}_B^H = \overline{V}_B^H.$$

(A19) implies

$$\mathcal{T}_S(V)(\cdot) \le \beta_S \left(1 - \overline{W}_B^H\right) + \beta_B \overline{W}_S^H = \overline{V}_S^H.$$

Parallel arguments show that $W_B(\cdot) \le \overline{W}_B^L$, $W_S(\cdot) \ge \overline{W}_S^L$, $\mathcal{T}_B(V)(\cdot) \le \overline{V}_B^L$, and $\mathcal{T}_S(V)(\cdot) \ge \overline{V}_S^L$. Therefore, the unique fixed point of $\mathcal{T}$ is in $\mathcal{B}'''$, and hence satisfies (A36) and (A37). ■

A5 Convergence

Although in the current model meeting partners cannot observe each other's market times, in the proof of our convergence result it will be useful to consider the (fictitious) updating factors if they could.

Let

$$\xi_B^{T_S}(t) \equiv \frac{f_S^L(t)}{f_S^H(t)}, \quad \xi_S^{T_B}(t) \equiv \frac{f_B^H(t)}{f_B^L(t)}, \tag{A38}$$

that is, $\xi_B^{T_S}(t)$ ($\xi_S^{T_B}(t)$) is the factor a buyer (seller) would use to update his optimism level from an observation of his partner's market time t, if he were able to observe such information.

As in the main text, for $n = 1, 2, \ldots$ and $t_B, t_S \ge 0$, we define

$$\xi_B^k(t_B, t_S) \equiv \xi_B^{k-1}(0,0)\xi_B^T(t_B)\xi_B^{T_S}(t_S), \tag{A39}$$

$$\xi_S^k(t_S, t_B) \equiv \xi_S^{k-1}(0,0)\xi_S^T(t_S)\xi_S^{T_B}(t_B), \tag{A40}$$

where

$$\xi_B^0(t_B, t_S) \equiv \xi_B^0, \quad \xi_S^0(t_S, t_B) \equiv \xi_S^0.$$

As in the main text, since market times are exponentially distributed, $\xi_B^0 \xi_B^T(t_{B1})\xi_B^{T_S}(t_{S1}) \cdots \xi_B^T(t_{Bn})\xi_B^{T_S}(t_{Sn})$ depends on $(t_{B1}, t_{S1}, \ldots, t_{Bn}, t_{Sn})$ only through n, $t_B = \sum_{i=1}^n t_{Bi}$, and $t_S = \sum_{i=1}^n t_{Si}$; in particular,

$$\xi_B^0 \xi_B^T(t_{B1})\xi_B^{T_S}(t_{S1}) \cdots \xi_B^T(t_{Bn})\xi_B^{T_S}(t_{Sn}) = \xi_B^n(t_{B1} + \cdots + t_{Bn}, t_{S1} + \cdots + t_{Sn}).$$

Similarly,

$$\xi_S^0 \xi_S^T(t_{S1})\xi_S^{T_B}(t_{B1})\cdots\xi_S^T(t_{Sn})\xi_S^{T_B}(t_{Bn}) = \xi_S^n(t_{S1}+\cdots+t_{Sn}, t_{B1}+\cdots+t_{Bn}).$$

Lemma A2. *(a) For any* $z \geq 0$, $\xi_B^{T_S}(\underline{t}_S(z)) \leq \xi_B^Z(z) \leq \xi_B^{T_S}(\bar{t}_S(z))$ and $\xi_S^{T_B}(\underline{t}_B(z)) \leq \xi_S^Z(z) \leq \xi_S^{T_B}(\bar{t}_B(z))$. *(b) For any* $t_B, t_S \geq 0$, $\pi_B^L(\xi_B^1(t_B, t_S)) = \pi_S^L(\xi_S^1(t_S, t_B))$ *and* $\pi_B^L(\xi_B^n(t_B, t_S)) > \pi_S^L(\xi_S^n(t_S, t_B))$ *when* $n = 2, 3 \ldots$.

Proof. Note that part (b) is the same as Lemma 2 in the main text. So we only need to prove part (a) as follows. The affiliation (Lemma A1) implies, for any $z \geq 0$,

$$\begin{aligned}\Pr(\tilde{\omega} = H|Z_B = z) &= \Pr(\tilde{\omega} = H|Z_B = z,\ \underline{t}_B(z) \leq T_B \leq \bar{t}_B(z)) \\ &\leq \Pr(\tilde{\omega} = H|Z_B = z,\ T_B = \bar{t}_B(z)) = \Pr(\tilde{\omega} = H|T_B = \bar{t}_B(z)).\end{aligned}$$

Therefore, $\xi_S^Z(z) \leq \xi_S^{T_B}(\bar{t}_B(z))$. One can similarly prove $\xi_S^Z(z) \geq \xi_S^{T_B}(\underline{t}_B(z))$ and $\xi_B^{T_S}(\underline{t}_S(z)) \leq \xi_B^Z(z) \leq \xi_B^{T_S}(\bar{t}_S(z))$ for any $z \geq 0$. It proves part (a). ■

Let T_B and T_S be independent (conditional on ω) random variables that follow the distributions $F_B^\omega(\cdot)$ and $F_S^\omega(\cdot)$ respectively in state ω. Let Z_B be an independent (conditional on T_B) random variable that follows the distribution $G_B(\cdot|T_B)$. Let Z_S be an independent (conditional on T_S) random variable that follows the distribution $G_S(\cdot|T_S)$. In an equilibrium-path meeting, the buyer's market time can be regarded as the random variable T_B, and the seller's market time can be regarded as the random variable T_S. The signals about the two market times can be regarded as the random variables Z_B, Z_S. The equilibrium price, regarded as a random variable, is either $p_B(Z_B, Z_S)$ if the buyer proposes, or $p_S(Z_S, Z_B)$ if the seller proposes.

For $i = 1, 2, \ldots$, let T_{Bi} and T_{Si} be independent (conditional on ω) random copies of T_B and T_S respectively. The following lemma generalizes Lemma 3 in the main text.

Lemma A3 (Belief convergence)**.** *There exist constants* $c_1, c_2 > 0$ *not depending on* r, δ, n *such that, in any full trade equilibrium under any friction profile* $(r, \delta) \in \mathbb{R}_+ \times \mathbb{R}_{++}$,

$$\mathbb{E}\left[\max_{k\in\{1,\ldots,n\}} \pi_B^L\left(\xi_B^k\left(\frac{T_B}{\theta^2}, \theta^2\sum_{i=1}^n T_{Si}\right)\right)|\omega = H\right] \leq (c_1 + c_2 n)\cdot\delta \quad \forall n = 1, 2, \ldots, \tag{A41}$$

$$\mathbb{E}\left[\max_{k\in\{1,\ldots,n\}} \pi_S^H\left(\xi_S^k\left(\frac{T_S}{\theta^2}, \theta^2\sum_{i=1}^n T_{Bi}\right)\right)|\omega = L\right] \leq (c_1 + c_2 n)\cdot\delta \quad \forall n = 1, 2, \ldots. \tag{A42}$$

Proof. We provide the proof of (A41) only. Using (A39), (A9), (A38), and (A2), we have, for $k = 1, \dots, n$,

$$\xi_B^k \left(\frac{T_B}{\theta^2}, \theta^2 \sum_{i=1}^n T_{Si} \right) = \xi_B^0 \left(\frac{\mu}{\alpha_B^H} \frac{\delta + \alpha_S^L}{\delta + \mu} \right)^k e^{-(\mu - \alpha_B^H) T_B / \theta^2} e^{(\mu - \alpha_S^L)\theta^2 (T_{S1} + \cdots + T_{Sn})}. \tag{A43}$$

Let

$$a \equiv \frac{\lambda_B^H}{\lambda_S^H} \frac{\lambda_S^L}{\lambda_S^L - \lambda_B^L}.$$

(A1) implies $a > 1$. Using (A3) and (A7),

$$\frac{\mu}{\alpha_B^H} \frac{\delta + \alpha_S^L}{\delta + \mu} \leq \frac{\mu}{\alpha_B^H(\delta + \mu)} \left(\delta + \frac{\delta \lambda_B^L}{\lambda_S^L - \lambda_B^L} \right) = \frac{(\delta + \mu)\lambda_B^H - \mu \lambda_S^H}{(\delta + \mu)\lambda_S^H} \frac{\lambda_S^L}{\lambda_S^L - \lambda_B^L} \leq \frac{\lambda_B^H}{\lambda_S^H} \frac{\lambda_S^L}{\lambda_S^L - \lambda_B^L} = a.$$

It follows that

$$\left(\frac{\mu}{\alpha_B^H} \frac{\delta + \alpha_S^L}{\delta + \mu} \right)^k \leq a^n$$

no matter whether the above LHS is greater than or smaller than 1. Then (A43) implies

$$\xi_B^k \left(\frac{T_B}{\theta^2}, \theta^2 \sum_{i=1}^n T_{Si} \right) \leq \xi_B^0 a^n e^{-(\mu - \alpha_B^H) T_B / \theta^2} e^{(\mu - \alpha_S^L)\theta^2 (T_{S1} + \cdots + T_{Sn})}.$$

Recall that, in state H, the density of T_B is $(\delta + \alpha_B^H) e^{-(\delta + \alpha_B^H) t_B}$, which is smaller than $(\delta + \alpha_B^H) e^{-(\delta + \alpha_B^H) t_B / \theta^2}$. Thus, for any $t_S \geq 0$,

$$\mathbb{E} \left[\max_{k \in \{1, \dots, n\}} \pi_B^L \left(\xi_B^k \left(\frac{T_B}{\theta^2}, \theta^2 \sum_{i=1}^n T_{Si} \right) \right) | T_{S1} + \cdots + T_{Sn} = t_S, \omega = H \right]$$
$$\leq \int_0^\infty \frac{\xi_B^0 a^n e^{-(\mu - \alpha_B^H) t_B / \theta^2} e^{(\mu - \alpha_S^L)\theta^2 t_S}}{1 + \xi_B^0 a^n e^{-(\mu - \alpha_B^H) t_B / \theta^2} e^{(\mu - \alpha_S^L)\theta^2 t_S}} (\delta + \alpha_B^H) e^{-(\delta + \alpha_B^H) t_B / \theta^2} dt_B. \tag{A44}$$

The last line can be rewritten as

$$\frac{\theta^2 (\delta + \alpha_B^H)}{\delta + \mu} \int_0^\infty \frac{\xi_B^0 a^n \frac{\delta + \mu}{\theta^2} e^{-(\delta + \mu) t_B / \theta^2} e^{(\mu - \alpha_S^L)\theta^2 t_S}}{1 + \xi_B^0 a^n e^{-(\delta + \mu) t_B / \theta^2} e^{(\delta + \alpha_B^H) t_B / \theta^2} e^{(\mu - \alpha_S^L)\theta^2 t_S}} dt_B. \tag{A45}$$

Using the fact that $e^{(\delta+\alpha_B^H)t_B/\theta^2} \geq 1$, the integral in (A45) is bounded from above by

$$\begin{aligned}
&\int_0^\infty \frac{\xi_B^0 a^n \frac{\delta+\mu}{\theta^2} e^{-(\delta+\mu)t_B/\theta^2} e^{(\mu-\alpha_S^L)\theta^2 t_S}}{1+\xi_B^0 a^n e^{-(\delta+\mu)t_B/\theta^2} e^{(\mu-\alpha_S^L)\theta^2 t_S}} dt_B \\
&= -\int_0^\infty \frac{d}{dt_B} \ln\left(1+\xi_B^0 a^n e^{-(\delta+\mu)t_B/\theta^2} e^{(\mu-\alpha_S^L)\theta^2 t_S}\right) dt_B \\
&= \ln\left(1+\xi_B^0 a^n e^{(\mu-\alpha_S^L)\theta^2 t_S}\right) \\
&\leq \ln(1+\xi_B^0) + n \ln a + \mu\theta^2 t_S. \qquad \text{(A46)}
\end{aligned}$$

Therefore, the conditional expected value in (A44) is bounded from above by

$$\frac{\theta^2(\delta+\alpha_B^H)}{\delta+\mu}\left(\ln(1+\xi_B^0) + n\ln a + \mu\theta^2 t_S\right),$$

which is linear in t_S. Replacing t_S with the random variable $T_{S1}+\cdots+T_{Sn}$ and taking expectation conditional on $\omega = H$, we have

$$\mathbb{E}\left[\max_{k\in\{1,\ldots,n\}} \pi_B^L\left(\xi_B^k\left(\frac{T_B}{\theta^2}, \theta^2\sum_{i=1}^n T_{Si}\right)\right)|\omega = H\right] \leq \frac{\theta^2(\delta+\alpha_B^H)}{\delta+\mu}\left(\ln(1+\xi_B^0)+n\ln a+\theta^2 n\right)$$

since $\mathbb{E}[T_{S1}+\cdots+T_{Sn}|\omega = H] = n\mathbb{E}[T_S|\omega = H] = \frac{n}{\delta+\mu} \leq \frac{n}{\mu}$. It together with (A6) implies

$$\begin{aligned}
&\mathbb{E}\left[\max_{k\in\{1,\ldots,n\}} \pi_B^L\left(\xi_B^k\left(\frac{T_B}{\theta^2}, \theta^2\sum_{i=1}^n T_{Si}\right)\right)|\omega = H\right] \\
&\leq \frac{\theta^2(\delta+\alpha_B^H)}{\mu}\left(\ln(1+\xi_B^0)+(\theta^2+\ln a)n\right) \\
&\leq \frac{\theta^2}{\mu}\left(\delta + \frac{\delta\lambda_S^H}{\lambda_B^H-\lambda_S^H}\right)\left(\ln(1+\xi_B^0)+(\theta^2+\ln a)n\right) \\
&= \delta\frac{\theta^2\lambda_B^H}{\mu(\lambda_B^H-\lambda_S^H)}\left[\ln\left(1+\frac{\phi^L\lambda_B^L}{\phi^H\lambda_B^H}\right)+\left(\theta^2+\ln\left(\frac{\lambda_B^H}{\lambda_S^H}\frac{\lambda_S^L}{\lambda_S^L-\lambda_B^L}\right)\right)n\right].
\end{aligned}$$

Therefore, (A41) holds with

$$c_1 \equiv \frac{\theta^2\lambda_B^H}{\mu(\lambda_B^H-\lambda_S^H)}\ln\left(1+\frac{\phi^L\lambda_B^L}{\phi^H\lambda_B^H}\right) > 0,$$

$$c_2 \equiv \frac{\theta^2 \lambda_B^H}{\mu(\lambda_B^H - \lambda_S^H)} \left[\theta^2 + \ln\left(\frac{\lambda_B^H}{\lambda_S^H} \frac{\lambda_S^L}{\lambda_S^L - \lambda_B^L}\right)\right] > 0.$$

■

Now comes the key proposition, which is the counterpart of Proposition 4 in the main text.

Proposition A3 (Convergence of prices to no uncertainty benchmarks)**.** *There exists some constant C not depending on r, δ such that, in any full trade equilibrium under any friction profile $(r, \delta) \in \mathbb{R}_+ \times \mathbb{R}_{++}$,*

$$\max \left\{ \begin{array}{c} \overline{p}_S^H - \mathbb{E}\left[p_S(Z_S, Z_B)|\omega = H\right], \\ \overline{p}_B^H - \mathbb{E}[p_B(Z_B, Z_S)|\omega = H], \\ \mathbb{E}\left[p_S(Z_S, Z_B)|\omega = L\right] - \overline{p}_S^L, \\ \mathbb{E}\left[p_B(Z_B, Z_S)|\omega = L\right] - \overline{p}_B^L \end{array} \right\} \leq C \cdot \delta.$$

It together with (A37), (A14), and (A15) implies that, with the discount rate r fixed, the expected discrepancy between the equilibrium transaction prices (i.e., $p_B(Z_B, Z_S)$ and $p_S(Z_S, Z_B)$) and the true-state certainty benchmark prices (i.e., $\overline{p}_B^L$, $\overline{p}_S^L$ in state L and $1 - \overline{p}_B^H$, $1 - \overline{p}_S^H$ in state H) is at most of order δ.

We will provide the proof of Proposition A3 for state H only; the proof for state L is parallel. As in the main text, we recursively and geometrically bounds the expected discrepancy between off-equilibrium search values and their no-uncertainty counterparts. Its formal statement requires the following definitions.

Let

$$\Delta^H W_B(\cdot) \equiv W_B(\cdot) - \overline{W}_B^H, \qquad \Delta^H W_S(\cdot) \equiv \overline{W}_S^H - W_S(\cdot),$$

$$\Delta^H V_B(\cdot) \equiv V_B(\cdot) - \overline{V}_B^H, \qquad \Delta^H V_S(\cdot) \equiv \overline{V}_S^H - V_S(\cdot).$$

From Proposition A2 these are all nonnegative; their monotonicity properties are also inherited from those of $W_B(\cdot), W_S(\cdot), V_B(\cdot), V_S(\cdot)$. For $n = 1, 2, \ldots$ and $t_B, t_S \geq 0$, define

$$\Pi_B^n(t_B, t_S) \equiv \max_{k \in \{1, \ldots, n\}} \pi_B^L \left(\xi_B^k \left(\frac{t_B}{\theta^2}, \theta^2 t_S\right)\right),$$

$$\hat{\Pi}_n \equiv \iint \cdots \int \Pi_B^n(t_{Bn}, t_{S1} + \cdots + t_{Sn}) dF_B^H(t_{Bn}) dF_S^H(t_{S1}) \cdots dF_S^H(t_{Sn}),$$

$$\hat{W}_S^1(t_B, t_S) \equiv \Delta^H W_S(\xi_S^1(\theta^2 t_S, t_B/\theta^2), t_S),$$

$$\hat{W}_B^n(t_B, t_S) \equiv \Delta^H W_B(\xi_B^n(t_B/\theta^2, \theta^2 t_S), t_B/\theta^2),$$

$$Q_B^n(t_B,t_S) \equiv \left(\beta_B \max\{\hat{W}_S^1, \hat{W}_B^1, \dots, \hat{W}_B^n\} + \beta_S \max\{\hat{W}_B^1, \dots, \hat{W}_B^n\}\right)(t_B,t_S),$$

$$\hat{Q}_n \equiv \iint \cdots \int Q_B^n(t_{Bn}; t_{S1} + \cdots + t_{Sn}) dF_B^H(t_{Bn}) dF_S^H(t_{S1}) \cdots dF_S^H(t_{Sn}).$$

Lemma A4. There is some $\hat{m} \in (0,1)$, not depending on r, δ, such that $\hat{Q}_n \leq \hat{\Pi}_n + \hat{m}\hat{Q}_{n+1}$ for all $n = 1, 2, \dots$.

Proof. Since $\hat{Q}_n$ involves $\hat{W}_S^1, \hat{W}_B^1, \dots, \hat{W}_B^n$, we derive a bound for $\hat{W}_S^1$ first. Take any $(\xi, t) \in \overline{\mathbb{R}}_+ \times \mathbb{R}_+$. Using $V_S(\cdot) \geq 0$, the monotonicity properties of $V_S(\cdot)$, and that the supports of $G_B(\cdot|t), G_S(\cdot|t)$ are $[\underline{z}_B(t), \bar{z}_B(t)], [\underline{z}_S(t), \bar{z}_S(t)]$, (A17) implies

$$\begin{aligned} W_S(\xi,t) &\geq \pi_S^H(\xi) m_S^H \iiint e^{-rt'_S} V_S(\xi\xi_S^T(t'_S), t+t'_S; z'_S, z'_B) dG_B^H(z'_B) dG_S(z'_S|t+t'_S) dF_S^H(t'_S) \\ &\geq \pi_S^H(\xi) m_S^H \iint e^{-rt'_S} V_S(\xi\xi_S^T(t'_S), t+t'_S; \bar{z}_S(t+t'_S), \underline{z}_B(t'_B)) dF_B^H(t'_B) dF_S^H(t'_S). \end{aligned}$$

Using $m_S^L, V_S(\cdot) \leq 1$, (A31) with $\omega = H$ implies

$$\overline{W}_S^H \leq \pi_S^L(\xi) + \pi_S^H(\xi) m_S^H \int e^{-rt'_S} \overline{V}_S^H dF_S^H(t'_S).$$

These, together with Proposition A2, imply

$$\begin{aligned} \Delta^H W_S(\xi,t) &\leq \pi_S^L(\xi) + \pi_S^H(\xi) m_S^H \iint e^{-rt'_S} \Delta^H V_S(\xi\xi_S^T(t'_S), t+t'_S; \theta(t+t'_S), t'_B/\theta) dF_B^H(t'_B) dF_S^H(t'_S) \\ &\leq \pi_S^L(\xi) + \iint \Delta^H V_S(\xi\xi_S^T(t'_S), t+t'_S; \bar{z}_S(t+t'_S), \underline{z}_B(t'_B)) dF_B^H(t'_B) dF_S^H(t'_S). \end{aligned} \tag{A47}$$

Using (A19), (A21), (A23), (A12), (A14), the monotonicity properties of W_B, W_S, ξ_S^T, the boundedness properties of $\underline{z}_B, \bar{z}_S$, and Lemma A2(a), we have

$$\begin{aligned} &V_S(\xi_S, t_S; z_S, z_B) \\ &\geq \beta_B p_B(z_B, z_S) + \beta_S p_S(z_S, z_B) \\ &= \beta_B W_S(\xi_S^0 \xi_S^T(\underline{t}_S(z_S)) \xi_S^Z(z_B), \underline{t}_S(z_S)) + \beta_S(1 - W_B(\xi_B^0 \xi_B^T(\underline{t}_B(z_B)) \xi_B^Z(z_S), \underline{t}_B(z_B))) \\ &\geq \beta_B W_S(\xi_S^0 \xi_S^T(\theta z_S) \xi_S^{T_B}(z_B/\theta), \underline{t}_S(z_S)) + \beta_S(1 - W_B(\xi_B^0 \xi_B^T(z_B/\theta) \xi_B^{T_S}(\theta z_S), \underline{t}_B(z_B))) \\ &= \beta_B W_S(\xi_S^1(\theta z_S, z_B/\theta), \underline{t}_S(z_S)) + \beta_S(1 - W_B(\xi_B^1(z_B/\theta, \theta z_S), \underline{t}_B(z_B))). \end{aligned}$$

Subtracting it from (A33) with $\omega = H$, we have

$$\Delta^H V_S(\xi_S, t_S; z_S, z_B) \le \beta_B \Delta^H W_S(\xi_S^1(\theta z_S, z_B/\theta), \underline{t}_S(z_S)) + \beta_S \Delta^H W_B(\xi_B^1(z_B/\theta, \theta z_S), \underline{t}_B(z_B)). \tag{A48}$$

Taking $\xi = \xi_S^1(\theta^2 \sum_{i=1}^n t_{Si}, t_{Bn}/\theta^2)$ and $t = \sum_{i=1}^n t_{Si}$ and $(t'_B, t'_S) = (t_{B(n+1)}, t_{S(n+1)})$ in (A47), and then using (A48), Lemma A2(b), the boundedness properties of $\bar{z}_S, \underline{z}_B$, and that $\underline{t}_S$ is the inverse of $\bar{z}_S$, we can bound $\hat{W}_S^1(t_{Bn}, \sum_{i=1}^n t_{Si})$ as follows:

$$\begin{aligned}
&\hat{W}_S^1\left(t_{Bn}, \sum_{i=1}^n t_{Si}\right) = \Delta^H W_S\left(\xi_S^1\left(\theta^2 \sum_{i=1}^n t_{Si}, \frac{t_{Bn}}{\theta^2}\right), \sum_{i=1}^n t_{Si}\right) \\
&\le \pi_S^L\left(\xi_S^1\left(\theta^2 \sum_{i=1}^n t_{Si}, \frac{t_{Bn}}{\theta^2}\right)\right) + \iint \left(\begin{array}{l} \beta_B \Delta^H W_S\left(\xi_S^1\left(\theta^2 \sum_{i=1}^{n+1} t_{Si}, \frac{t_{B(n+1)}}{\theta^2}\right), \sum_{i=1}^{n+1} t_{Si}\right) \\ +\beta_S \Delta^H W_B\left(\xi_B^1\left(\frac{t_{B(n+1)}}{\theta^2}, \theta^2 \sum_{i=1}^{n+1} t_{Si}\right), \frac{t_{B(n+1)}}{\theta^2}\right) \end{array}\right) \begin{array}{l} dF_B^H(t_{B(n+1)}) \\ dF_S^H(t_{S(n+1)}) \end{array} \\
&= \pi_B^L\left(\xi_B^1\left(\frac{t_{Bn}}{\theta^2}, \theta^2 \sum_{i=1}^n t_{Si}\right)\right) + \iint (\beta_B \hat{W}_S^1 + \beta_S \hat{W}_B^1)\left(t_{B(n+1)}, \sum_{i=1}^{n+1} t_{Si}\right) dF_B^H(t_{B(n+1)}) dF_S^H(t_{S(n+1)}) \\
&\le \Pi_B^n\left(t_{Bn}, \sum_{i=1}^n t_{Si}\right) + \iint Q_{n+1}\left(t_{B(n+1)}, \sum_{i=1}^{n+1} t_{Si}\right) dF_B^H(t_{B(n+1)}) dF_S^H(t_{S(n+1)}).
\end{aligned} \tag{A49}$$

Next, we derive bounds for $\hat{W}_B^1, \ldots, \hat{W}_B^n$. Take any $(\xi, t) \in \overline{\mathbb{R}}_+ \times \mathbb{R}_+$. Let $\tilde{\omega}, T_B, T_S, Z_B, Z_S$ be random variables distributed according to $\Pr(\tilde{\omega} = \omega) = \pi_B^\omega(\xi)$, $T_B|\tilde{\omega} \sim F_B^{\tilde{\omega}}(\cdot)$, $T_S|\tilde{\omega} \sim F_S^{\tilde{\omega}}(\cdot)$, $Z_B|(t + T_B) \sim G_B(\cdot|t + T_B)$, and $Z_S|T_S \sim G_S(\cdot|T_S)$. Then (A22) implies

$$\begin{aligned}
&\sum_\omega \pi_B^\omega(\xi) m_B^\omega \iiiint e^{-rt'_B} P_B(\xi \xi_B^T(t'_B), t + t'_B; z'_B, z'_S) dG_S(z'_S|t'_S) dF_S^\omega(t'_S) dG_B(z'_B|t + t'_B) dF_B^\omega(t'_B) \\
&= \mathbb{E}\left[m_B^{\tilde{\omega}} e^{-rT_B} P_B(\xi \xi_B^T(T_B), t + T_B; Z_B, Z_S)\right] \\
&= \mathbb{E}\left[\mathbb{E}\left[m_B^{\tilde{\omega}}|T_B, Z_B, Z_S\right] e^{-rT_B} P_B(\xi \xi_B^T(T_B), t + T_B; Z_B, Z_S)\right] \\
&\le \mathbb{E}\left[\begin{array}{l} \mathbb{E}\left[m_B^{\tilde{\omega}}|T_B, Z_B, Z_S\right] e^{-rT_B} \times \\ \mathbb{E}\left[\max\left\{1 - \rho_S(T_S, Z_B\}, W_B(\xi \xi_B^T(T_B) \xi_B^Z(Z_S), t + T_B)\right\}|T_B, Z_B, Z_S\right] \end{array}\right].
\end{aligned}$$

Since $m_B^L > m_B^H$ and $1 - \rho_S(\cdot, z_B)$ is increasing, and $\tilde{\omega}, -T_S$ conditional on T_B, Z_B, Z_S are affiliated (provided we declare $L < H$), the above last line is at most

$$\begin{aligned}
\mathbb{E}&\left[\mathbb{E}\left[m_B^{\tilde{\omega}} e^{-rT_B} \max\left\{1-\rho_S(T_S,Z_B), W_B(\xi\xi_B^T(T_B)\xi_B^Z(Z_S), t+T_B)\right\} | T_B, Z_B, Z_S\right]\right] \\
&= \mathbb{E}\left[m_B^{\tilde{\omega}} e^{-rT_B} \max\left\{1-\rho_S(T_S,Z_B), W_B(\xi\xi_B^T(T_B)\xi_B^Z(Z_S), t+T_B)\right\}\right] \\
&= \sum_{\omega} \pi_B^{\omega}(\xi) m_B^{\omega} \iiiint e^{-rt'_B} \max\left\{1-\rho_S(t'_S, z'_B), W_B(\xi\xi_B^T(t'_B)\xi_B^Z(z'_S), t+t'_B)\right\} \\
&\qquad\qquad dG_S(z'_S|t'_S) dF_S^{\omega}(t'_S) dG_B(z'_B|t+t'_B) dF_B^{\omega}(t'_B).
\end{aligned}$$

This together with (A16), (A18), (A20), (A13), and (A15) implies

$$\begin{aligned}
W_B(\xi,t) \le \pi_B^L(\xi) + m_B^H \iiiint e^{-rt'_B} &[\beta_B \max\left\{1 - W_S(\xi_S^0 \xi_S^T(t'_S)\xi_S^Z(z'_B), t'_S), W_B(\xi\xi_B^T(t'_B)\xi_B^Z(z'_S), t+t'_B)\right\} \\
&+ \beta_S \max\left\{W_B(\xi_B^0\xi_B^T(z'_B)\xi_B^Z(z'_S), z'_B), W_B(\xi\xi_B^T(t'_B)\xi_B^Z(z'_S), t+t'_B)\right\}] \\
&\qquad dG_S(z'_S|t'_S) dF_S^H(t'_S) dG_B(z'_B|t+t'_B) dF_B^H(t'_B).
\end{aligned}$$

Using the monotonicity properties of W_B, W_S, ξ_B^Z, ξ_S^Z, ξ_B^T, ξ_S^T, Lemma A2(a), that the supports of $G_B(\cdot|t+t'_B)$, $G_S(\cdot|t'_S)$ are $[\underline{z}_B(t+t'_B), \bar{z}_B(t+t'_B)]$, $[\underline{z}_S(t'_S), \bar{z}_S(t'_S)]$, and the boundedness properties of $\underline{z}_B, \bar{z}_S, \underline{t}_B, \bar{t}_S$, the elements in the above integrand can be bounded as follows:

$$W_S(\xi_S^0\xi_S^T(t'_S)\xi_S^Z(z'_B), t'_S) \ge W_S\left(\xi_S^0\xi_S^T(t'_S)\xi_S^Z\left(\frac{t+t'_B}{\theta}\right), t'_S\right) \ge W_S\left(\xi_S^0\xi_S^T(\theta^2 t'_S)\xi_S^{T_B}\left(\frac{t'_B}{\theta^2}\right), t'_S\right),$$

$$W_B(\xi\xi_B^T(t'_B)\xi_B^Z(z'_S), t+t'_B) \le W_B(\xi\xi_B^T(t'_B)\xi_B^Z(\theta t'_S), t'_B) \le W_B\left(\xi\xi_B^T\left(\frac{t'_B}{\theta^2}\right)\xi_B^{T_S}(\theta^2 t'_S), \frac{t'_B}{\theta^2}\right),$$

$$W_B(\xi_B^0\xi_B^T(z'_B)\xi_B^Z(z'_S), z'_B) \le W_B\left(\xi_B^0\xi_B^T\left(\frac{t+t'_B}{\theta}\right)\xi_B^Z(\theta t'_S), \frac{t+t'_B}{\theta}\right) \le W_B\left(\xi_B^0\xi_B^T\left(\frac{t'_B}{\theta^2}\right)\xi_B^{T_S}(\theta^2 t'_S), \frac{t'_B}{\theta^2}\right).$$

Using (A30) and (A29) with $\omega = H$, we have

$$\overline{W}_B^H = m_B^H \int e^{-rt'_B}\left[\beta_B \max\{1-\overline{W}_S^H, \overline{W}_B^H\} + \beta_S \overline{W}_B^H\right] dF_B^H(t'_B).$$

Combining the results and using $e^{-rt'_B} \leq 1$, we have

$$\Delta^H W_B(\xi,t) \leq \pi_B^L(\xi) + m_B^H \iint \left[\beta_B \max\left\{\Delta^H W_S\left(\xi_S^1\left(\theta^2 t'_S, \frac{t'_B}{\theta^2}\right), t'_S\right), \Delta^H W_B\left(\xi\xi_B^T\left(\frac{t'_B}{\theta^2}\right)\xi_B^{T_S}(\theta^2 t'_S), \frac{t'_B}{\theta^2}\right)\right\}\right.$$
$$\left. + \beta_S \max\left\{\Delta^H W_B\left(\xi_B^1\left(\frac{t'_B}{\theta^2}, \theta^2 t'_S\right), \frac{t'_B}{\theta^2}\right), \Delta^H W_B\left(\xi\xi_B^T\left(\frac{t'_B}{\theta^2}\right)\xi_B^{T_S}(\theta^2 t'_S), \frac{t'_B}{\theta^2}\right)\right\}\right] dF_B^H(t'_B) dF_S^H(t'_S). \tag{A50}$$

Taking $k = 1, 2, \ldots, n$, $\xi = \xi_B^k(t_{Bn}/\theta^2, \theta^2 \sum_{i=1}^n t_{Si})$, $t = t_{Bn}/\theta^2$ and $(t'_B, t'_S) = (t_{B(n+1)}, t_{S(n+1)})$ in (A50), we can bound $\hat{W}_B^k(t_{Bn}, \sum_{i=1}^n t_{Si})$ as follows:

$$\begin{aligned}
&\hat{W}_B^k\left(t_{Bn}, \sum_{i=1}^n t_{Si}\right) \\
&= \Delta^H W_B\left(\xi_B^k\left(\frac{t_{Bn}}{\theta^2}, \theta^2 \sum_{i=1}^n t_{Si}\right), \frac{t_{Bn}}{\theta^2}\right) \\
&\leq \pi_B^L\left(\xi_B^k\left(\frac{t_{Bn}}{\theta^2}, \theta^2 \sum_{i=1}^n t_{Si}\right)\right) \\
&\quad + m_B^H \iint \left(\begin{array}{c} \beta_B \max\left\{\begin{array}{l} \Delta^H W_S\left(\xi_S^1\left(\theta^2 \sum_{i=1}^{n+1} t_{Si}, \frac{t_{B(n+1)}}{\theta^2}\right), \sum_{i=1}^{n+1} t_{Si}\right), \\ \Delta^H W_B\left(\xi_B^{k+1}\left(\frac{t_{B(n+1)}}{\theta^2}, \theta^2 \sum_{i=1}^{n+1} t_{Si}\right), \frac{t_{B(n+1)}}{\theta^2}\right) \end{array}\right\} \\ + \beta_S \max\left\{\begin{array}{l} \Delta^H W_B\left(\xi_B^1\left(\frac{t_{B(n+1)}}{\theta^2}, \theta^2 \sum_{i=1}^{n+1} t_{Si}\right), \frac{t_{B(n+1)}}{\theta^2}\right), \\ \Delta^H W_B\left(\xi_B^{k+1}\left(\frac{t_{B(n+1)}}{\theta^2}, \theta^2 \sum_{i=1}^{n+1} t_{Si}\right), \frac{t_{B(n+1)}}{\theta^2}\right) \end{array}\right\} \end{array}\right) \begin{array}{l} dF_B^H(t_{B(n+1)}) \\ dF_S^H(t_{S(n+1)}) \end{array} \\
&= \pi_B^L\left(\xi_B^k\left(\frac{t_{Bn}}{\theta^2}, \theta^2 \sum_{i=1}^n t_{Si}\right)\right) + m_B^H \iint \left(\begin{array}{c} \beta_B \max\{\hat{W}_S^1, \hat{W}_B^{k+1}\}(t_{B(n+1)}, \sum_{i=1}^{n+1} t_{Si}) \\ + \beta_S \max\{\hat{W}_B^1, \hat{W}_B^{k+1}\}(t_{B(n+1)}, \sum_{i=1}^{n+1} t_{Si}) \end{array}\right) \begin{array}{l} dF_B^H(t_{B(n+1)}) \\ dF_S^H(t_{S(n+1)}) \end{array} \\
&\leq \Pi_B^n\left(t_{Bn}, \sum_{i=1}^n t_{Si}\right) + m_B^H \iint Q_{n+1}\left(t_{B(n+1)}, \sum_{i=1}^{n+1} t_{Si}\right) dF_B^H(t_{B(n+1)}) dF_S^H(t_{S(n+1)}).
\end{aligned} \tag{A51}$$

Since (A51) is true for all $k = 1, 2, \ldots, n$, we have

$$\begin{aligned}
&\max\left\{\hat{W}_B^1, \ldots, \hat{W}_B^n\right\}(t_{Bn}, \sum_{i=1}^n t_{Si}) \\
&\qquad \leq \Pi_B^n\left(t_{Bn}, \sum_{i=1}^n t_{Si}\right) + m_B^H \iint Q_{n+1}\left(t_{B(n+1)}, \sum_{i=1}^{n+1} t_{Si}\right) dF_B^H(t_{B(n+1)}) dF_S^H(t_{S(n+1)}).
\end{aligned} \tag{A52}$$

Since $m_B^H \leq 1$, (A52) together with (A49) implies

$$\max\left\{\hat{W}_S^1, \hat{W}_B^1, \ldots, \hat{W}_B^n\right\}(t_{Bn}, \sum_{i=1}^n t_{Si})$$
$$\leq \Pi_B^n\left(t_{Bn}, \sum_{i=1}^n t_{Si}\right) + \iint Q_{n+1}\left(t_{B(n+1)}, \sum_{i=1}^{n+1} t_{Si}\right) dF_B^H(t_{B(n+1)}) dF_S^H(t_{S(n+1)}). \quad \text{(A53)}$$

Multiplying (A53) by β_B and (A52) by β_S, and summing the inequalities, we obtain

$$Q_B^n\left(t_{Bn}, \sum_{i=1}^n t_{Si}\right)$$
$$\leq \Pi_B^n\left(t_{Bn}, \sum_{i=1}^n t_{Si}\right) + (\beta_B + \beta_S m_B^H)\iint Q_{n+1}\left(t_{B(n+1)}, \sum_{i=1}^{n+1} t_{Si}\right) dF_B^H(t_{B(n+1)}) dF_S^H(t_{S(n+1)}). \quad \text{(A54)}$$

Note that

$$m_B^H = \frac{\mu \lambda_S^H}{(\delta + \mu)\lambda_B^H} \leq \frac{\lambda_S^H}{\lambda_B^H} < 1.$$

Let

$$\hat{m} \equiv \beta_B + \beta_S \frac{\lambda_S^H}{\lambda_B^H} < 1$$

so that

$$\beta_B + \beta_S m_B^H \leq \hat{m} < 1. \quad \text{(A55)}$$

Integrating (A54) with respect to $t_{Bn}, t_{S1}, \ldots, t_{Sn}$ using the c.d.f.'s $F_B^H, F_S^H, \ldots, F_S^H$, we obtain, for $n = 1, 2, \ldots$,

$$\hat{Q}_n \leq \hat{\Pi}_n + \hat{m}\hat{Q}_{n+1}. \quad \text{(A56)}$$

■

Now we are ready to prove Proposition A3.

Proof of Proposition A3. We provide the proof for state H here. First note that

$$\begin{aligned}
&\overline{p}_S^H - \mathbb{E}\left[p_S(Z_S, Z_B)|\omega = H\right] \\
&= \mathbb{E}\left[W_B(\xi_B^0 \xi_B^T(\underline{t}_B(Z_B))\xi_B^Z(Z_S), \underline{t}_B(Z_B))|\omega = H\right] - \overline{W}_B^H \\
&\leq \mathbb{E}\left[W_B(\xi_B^0 \xi_B^T(Z_B/\theta)\xi_B^{T_S}(\theta Z_S), Z_B/\theta)|\omega = H\right] - \overline{W}_B^H \\
&\leq \mathbb{E}\left[W_B(\xi_B^0 \xi_B^T(T_B/\theta^2)\xi_B^{T_S}(\theta^2 T_S), T_B/\theta^2)|\omega = H\right] - \overline{W}_B^H \\
&= \mathbb{E}\left[\hat{W}_B^1(T_B, T_S)|\omega = H\right]
\end{aligned}$$

and

$$\begin{aligned}
&\overline{p}_B^H - \mathbb{E}[p_B(Z_B, Z_S)|\omega = H] \\
&= \overline{W}_S^H - \mathbb{E}\left[W_S(\xi_S^0 \xi_S^T(\underline{t}_S(Z_S))\xi_S^Z(Z_B), \underline{t}_S(Z_S))|\omega = H\right] \\
&\leq \overline{W}_S^H - \mathbb{E}\left[W_S(\xi_S^0 \xi_S^T(\underline{t}_S(\bar{z}_S(T_S)))\xi_S^{T_B}(T_B/\theta), \underline{t}_S(\bar{z}_S(T_S)))|\omega = H\right] \\
&= \overline{W}_S^H - \mathbb{E}\left[W_S(\xi_S^0 \xi_S^T(T_S)\xi_S^{T_B}(T_B/\theta), T_S)|\omega = H\right] \\
&\leq \overline{W}_S^H - \mathbb{E}\left[W_S(\xi_S^0 \xi_S^T(\theta^2 T_S)\xi_S^{T_B}(T_B/\theta^2), T_S)|\omega = H\right] \\
&= \mathbb{E}\left[\hat{W}_S^1(T_B, T_S)|\omega = H\right].
\end{aligned}$$

Using (A52) and (A49),

$$\max\{\mathbb{E}[\hat{W}_B^1(T_B, T_S)|\omega = H], \mathbb{E}[\hat{W}_S^1(T_B, T_S)|\omega = H]\} \leq \hat{\Pi}_1 + \hat{Q}_2.$$

While $\hat{\Pi}_1 \leq (c_1 + c_2)\delta$ from Lemma A3, $\hat{Q}_2$ is, from Lemma A4, bounded by

$$\hat{Q}_2 \leq \sum_{n=2}^{\infty} \hat{m}^{n-2}\hat{\Pi}_n \leq \delta \sum_{n=2}^{\infty} \hat{m}^{n-2}(c_1 + c_2 n). \tag{A57}$$

It is well known that $0 < \hat{m} < 1$ implies that the series $\sum_n \hat{m}^n$ and $\sum_n n\hat{m}^n$ are convergent,[5] so that the series in (A57) is convergent. It proves that there exists a constant C, not depending on r, δ, such that

$$\max\{\mathbb{E}[\hat{W}_B^1(T_B, T_S)|\omega = H], \mathbb{E}[\hat{W}_S^1(T_B, T_S)|\omega = H]\} \leq C \cdot \delta$$

[5] Indeed, $\sum_{n=1}^{\infty} \hat{m}^n = \hat{m}/(1-\hat{m})$ and $\sum_{n=1}^{\infty} n\hat{m}^n = \hat{m}/(1-\hat{m})^2$.

as desired. ■

Corollary 1 (Convergence of prices to Walrasian levels)**.** *There exist constants $C_0, C_1 > 0$ not depending on r, δ such that, in any full trade equilibrium under any friction profile $(r, \delta) \in \mathbb{R}_+ \times \mathbb{R}_{++}$,*

$$\max \left\{ \begin{array}{c} 1 - \mathbb{E}\left[p_S(Z_S, Z_B)|\omega = H\right], \\ 1 - \mathbb{E}[p_B(Z_B, Z_S)|\omega = H], \\ \mathbb{E}\left[p_S(Z_S, Z_B)|\omega = L\right], \\ \mathbb{E}\left[p_B(Z_B, Z_S)|\omega = L\right] \end{array} \right\} \leq C_1 \cdot (r + \delta),$$

and when $r + \delta > 0$ is sufficiently small,

$$\min \left\{ \begin{array}{c} 1 - \mathbb{E}\left[p_S(Z_S, Z_B)|\omega = H\right], \\ 1 - \mathbb{E}[p_B(Z_B, Z_S)|\omega = H], \\ \mathbb{E}\left[p_S(Z_S, Z_B)|\omega = L\right], \\ \mathbb{E}\left[p_B(Z_B, Z_S)|\omega = L\right] \end{array} \right\} \geq C_0 \cdot (r + \delta).$$

That is, the expected discrepancy between the equilibrium transaction prices (i.e., $p_B(Z_B, Z_S)$ and $p_S(Z_S, Z_B)$) and the true-state Walrasian price (i.e., 0 in state L and 1 in state H) is of order $r + \delta$.

Proof. It follows from Proposition 1 in the main text, Proposition A2, and Proposition A3. ■

We thus conclude that the rate of convergence of the equilibrium transaction prices is the same as it would be if the true state were commonly known.

References

Milgrom, P. R., Weber, R. J., 1982. A theory of auctions and competitive bidding. Econometrica 50 (5), 1089–1122.