The Role of Private Information in Dynamic Matching and Bargaining: Can It Be Good for Efficiency?*

Artyom Shneyerov,†,‡ Adam Chi Leung Wong§,

*CIREQ, CIRANO and Department of Economics, Concordia University, 1455 de Maisonneuve Blvd. West, Montreal, Quebec H3G 1M8, Canada
School of International Business Administration, Shanghai University of Finance and Economics, 777 Guoding Road, Shanghai, China 200433

February 3, 2011

Abstract

We consider a private information replica of the dynamic matching and bargaining model of Mortensen and Wright (2002). We find that private information typically deters entry. But, the welfare can actually be higher under private information.

Keywords: Markets with search frictions, Matching and Bargaining, Two-sided Incomplete Information

JEL Classification Numbers: C73, C78, D83.

1 Introduction

Can private information improve the efficiency of bargaining? This is an important question for the design of bargaining institutions. In this letter, we show that it can if bargaining is embedded in a dynamic matching market. We do so by comparing equilibrium outcomes of two steady-state models of search and bargaining. The first is the full information take-it-or-leave-it bargaining model of Rubinstein and Wolinsky (1985, 1990), but with heterogeneous traders as in Gale (1986, 1987), and with a general matching technology as in Mortensen and Wright (2002). The second is the private information replica of Mortensen and Wright (2002), recently investigated in Shneyerov and Wong (2010a,b).

Mortensen and Wright (2002) and Shneyerov and Wong (2010a) have shown that for sufficiently small discount rate, in each model there is a unique full trade equilibrium: every

---

*The results appearing in this paper were originally contained in the 2006 version of Shneyerov and Wong (2010a). We thank the editor of this journal and an anonymous referee for their comments.
†Corresponding author. Tel.: +1 514 648 2424 ext 5288. Fax: +1 514 648 4536
‡E-mail address: achneero@alcor.concordia.ca
§E-mail address: wongchileung@gmail.com
meeting results in trade.\footnote{Satterthwaite and Shneyerov (2007) have shown existence of such equilibria in a dynamic matching model with auctions.} In this letter, we consider the case of a small discount factor, and therefore restrict attention to full trade equilibria.

In a full trade equilibrium, there is less entry relative to the frictionless, Walrasian benchmark; the marginal participating buyer type is higher than the Walrasian price, and the marginal participating seller type is lower. Therefore, the inefficiency arises only because of costly delay and insufficient entry. The cost of delay is due to the discounting and also, importantly, due to the explicit search cost.

When there is no discounting, the equilibrium outcomes in both models are the same, and hence welfare is also the same. When $r$ increases by a small amount $dr$, the cost of delay due to discounting increases in both models by the same marginal amount. The cost of search, however, is affected only under full information. Shneyerov and Wong (2010a) show that under private information, the marginal participating types do not change; the entry margin and the search cost are not affected. But under full information, the analysis in Mortensen and Wright (2002) implies that the marginal types move towards the Walrasian price, and hence there is more entry.

These entering traders increase the cost of search on their own side, at the same time reducing the cost of search on the other side. These effects are first-order in $dr$. Because the marginal types only make profit on the order of $dr$, and the entry margin itself is also of the order $dr$, the contribution of the entrants’ own welfare is of the second order.

We consider a constant returns to scale matching technology. The search cost incurred by a trader until the next meeting depends on equilibrium through the ratio $\zeta$ of buyers and sellers in the market (market tightness). The search cost is minimized at some $\zeta = \zeta^*$ that only depends on model primitives. Under private information, we show that the cost of search is minimized in a full trade equilibrium when the proposal probabilities satisfy the Hosios (1990) condition. Under full information, then, entry can either increase or decrease the search cost, depending on whether $\zeta$ moves farther away or closer to $\zeta^*$. The welfare under full information can be either higher or lower than under private information. We derive a precise condition for this effect.

In a model different from ours, with one-time entry of traders and costless search, the welfare-improving effect of private information was first noticed by Moreno and Wooders (2001). They provide a numerical example showing that welfare can be slightly higher under private information, but do not address this issue formally.

We are aware of only one other paper that contains formal results on the comparison of full and private information. Lauermann (2011) considers a model with no search cost, only with the exogenous exit of traders as in Satterthwaite and Shneyerov (2008). Moreover, sellers are homogeneous, with cost normalized to 0, and only they make offers. He finds that equilibria converge to efficiency only under private information, and therefore the welfare is large than under full information when frictions are small. This is because, under full information, the sellers always obtain certain rents from their offers, which are only acceptable to buyers with large enough values; whereby, inefficiency persists even in the limit. Although his message is similar, the nature of Lauermann (2011)’s model and results is very different from ours.
2 The Models

We begin by reviewing the models of Mortensen and Wright (2002) (hereafter MW) and Shneyerov and Wong (2010a), but focus only on the essential details. There are potential buyers and sellers of a homogeneous, indivisible good. Potential buyers and sellers are continuously born at rates $b > 0$ and $s > 0$. Each buyer wants at most 1 unit of the good, each seller has 1 unit. All traders are risk neutral. The types of new-born traders are drawn at the time of their birth and remain the same throughout their life in the market. Buyers draw their valuations $v \in [0,1]$ i.i.d. from distribution $F$ and the sellers draw their costs $c \in [0,1]$ i.i.d. from distribution $G$. We assume that $F$ and $G$ have densities on $[0,1]$ bounded away from 0 and $\infty$. Time is continuous and infinite horizon. The agents discount future utility at the instantaneous rate $r$.

Once born, each trader decides whether to enter the market. Those who do not enter get zero payoff. Those who enter incur the search cost continuously at the rates $\kappa_B > 0$ and $\kappa_S > 0$, until they leave the market.

We model the process of search by the means of a matching function $M(B,S)$ that gives the rate of matching as a function of the mass of active buyers $B$ and the mass of active sellers $S$. The function $M$ (i) is continuous on $\mathbb{R}_+^2$, (ii) is nondecreasing in each argument, (iii) satisfies $M(0,S) = M(B,0) = 0$, and (iv) exhibits constant returns to scale. Denote by $\zeta \equiv B/S$ the (steady-state) ratio of active buyers to sellers. The arrival rates for buyers and sellers are

$$
\ell_B(\zeta) \equiv M(\zeta,1)/\zeta, \quad \ell_S(\zeta) \equiv M(\zeta,1).
$$

Once a pair of buyer and seller is matched, they bargain either under full or private information. With probability $\alpha_S \in (0,1)$, the seller makes a final offer to the buyer, and with probability $\alpha_B = 1 - \alpha_S$, the buyer proposes and the seller responds. If a type $v$ buyer and a type $c$ seller trade at a price $p$, then they leave the market with payoffs $v-p$, and $p-c$ respectively. If the bargaining breaks down, both traders return to the pool of unmatched traders and search again.

Let $W_B(v)$ and $W_S(c)$ be traders’ equilibrium values of search. A buyer of type $v$ will accept any price offer $p$ if $v-p > W_B(v)$, and similarly any seller of type $c$ will accept any price offer $p$ if $p-c > W_S(c)$. The responding strategies are therefore characterized by the reservation prices $v-W_B(v)$ for the buyers and $c+W_S(c)$ for the sellers. The proposing strategies differ according to whether the information is full or private. Under full information, a type $v$ buyer and a type $c$ seller trade if and only if $v-c \geq W_S(c)+W_B(v)$, in which case the proposer proposes his partner’s reservation price, i.e. either $c+W_S(c)$ or $v-W_B(v)$. Under private information, the proposing strategies are chosen to maximize expected continuation payoffs given the market distributions of their partners’ reservation prices.

We focus on a simple class of nontrivial steady-state equilibria, full trade equilibria, in which every meeting results in trade. The only equilibrium components that will matter for us here are (i) $\underline{v}$ and $\bar{c}$, the marginal participating types of buyers and sellers

$$
\underline{v} \equiv \inf \{ v : W_B(v) > 0 \}, \quad \bar{c} \equiv \sup \{ c : W_S(c) > 0 \},
$$

and (ii) $\zeta$, the steady state ratio of the market stocks of buyers ($B$) and sellers ($S$). Shneyerov and Wong (2010a) prove several properties of a full trade equilibrium in the private infor-
mation model. First, the supports for active buyers’ types and active sellers’ types are separate, i.e. \( \underline{v} > \bar{c} \). This is because otherwise a buyer with \( \underline{v} \) will not trade if he meets a seller with \( \bar{c} \): the seller will not propose or accept anything less than \( \bar{c} \), while the buyer will only propose or accept something below \( \underline{v} \). Second, under private information, the lowest buyer’s (and hence all active buyers’) offer is exactly at the level just acceptable to all active sellers, i.e. is equal to \( \bar{c} \); and similarly, the highest seller’s (and hence all active sellers’) offer is exactly at the level just acceptable to all active buyers, i.e. equal to \( \underline{v} \).

Mortensen and Wright (2002) and Shneyerov and Wong (2010a) show that the Walrasian price \( p_W \), defined as the unique solution to \( sG(p) = b[1-F(p)] \), falls in between the marginal participating types: \( p_W \in (\bar{c}, \underline{v}) \).

We index the private (resp. full) information equilibrium objects by \( p \) (resp. by \( f \)). Under private information, the full trade equilibrium admits a very simple characterization. The marginal traders must be indifferent between participating or not, which leads to the following two indifference equations:

\[
\ell_B(\zeta_p)\alpha_B (\underline{v}_p - \bar{c}_p) = \kappa_B, \tag{2}
\]

\[
\ell_S(\zeta_p)\alpha_S (\underline{v}_p - \bar{c}_p) = \kappa_S. \tag{3}
\]

The intuition of (2) is that, first, the marginal buyers make positive profit only when they propose. Second, they propose \( \bar{c}_p \). Because marginal buyers make 0 expected net profit, their expected profit rate from bargaining, \( \ell_B(\zeta_p)\alpha_B (\underline{v}_p - \bar{c}_p) \), must be equal to \( \kappa_B \), the rate of their search cost. The intuition for (3) is parallel.

From (1), \( \ell_S(\zeta_p)/\ell_B(\zeta_p) = \zeta_p \), so (2) and (3) can be easily solved for \( \zeta_p \) and \( \underline{v}_p - \bar{c}_p \):

\[
\zeta_p = \frac{\alpha_B \kappa_S}{\alpha_S \kappa_B}, \tag{4}
\]

\[
\underline{v}_p - \bar{c}_p = K(\zeta_p), \tag{5}
\]

where

\[
K(\zeta) = \frac{\kappa_B}{\ell_B(\zeta)} + \frac{\kappa_S}{\ell_S(\zeta)}. \nonumber
\]

A necessary condition for existence of a full trade equilibrium is that \( K(\zeta_p) < 1 \), which we assume throughout this paper.

Since successful traders leave the market in matched pairs, in steady state, the inflow of active buyers must equal the inflow of active sellers:

\[
b[1-F(\underline{v}_p)] = sG(\bar{c}_p). \tag{6}
\]

Since \( \underline{v}_p - \bar{c}_p \) is determined from (5), \( \underline{v}_p \) and \( \bar{c}_p \) are uniquely pinned down by (6). Note that neither the marginal participating types nor \( \zeta_p \) depend on \( r \). The triple \( (\zeta_p, \underline{v}_p, \bar{c}_p) \) may not always characterize an equilibrium, because the marginal buyers may have an incentive to offer prices below \( \bar{c}_p \), and marginal sellers – to offer prices above \( \underline{v}_p \). However, Shneyerov and Wong (2010a, Proposition 3) show that when \( r \) is small these solutions do characterize an equilibrium and this equilibrium is unique. Further, the search values of buyers and sellers are given by

\[
W_{Bp}(v) = \frac{\ell_B(\zeta_p)}{r} (v - \underline{v}_p), \quad W_{Sp}(c) = \frac{\ell_S(\zeta_p)}{r} (\bar{c}_p - c). \tag{7}
\]
The total utility flow of the arriving buyers and sellers is

\[ W_p (r) = b \int_0^1 W_{Bp} (v) dF (v) + s \int_0^{\zeta_p} W_{Sp} (c) dG (c). \tag{8} \]

The characterization of a full trade equilibrium in the full information model is slightly more complicated because the payoffs of the marginal types depend on the market distribution of partner types, not only on the marginal partner type. Because each meeting results in trade, these distributions are truncations of the original distributions of types in the arriving flows, i.e., are equal to \( \frac{F(v)}{1-F(\zeta_f)} \) for buyers and \( \frac{G(c)}{G(\bar{c}_f)} \) for sellers. Now \( \zeta_f, \nu_f \) and \( \bar{c}_f \) are functions of \( r \) implicitly determined by the equations parallel to the private information model:

\[ \ell_B (\zeta_f) \alpha_B \int_0^{\nu_f} \frac{[\nu_f - c - W_{Sf} (c)]}{\bar{c}_f} dG (c) \frac{dG (c)}{G (\bar{c}_f)} = \kappa_B, \tag{9} \]

\[ \ell_S (\zeta_f) \alpha_S \int_0^{\nu_f} [v - W_{Bf} (v) - \nu_f] \frac{dF (v)}{1-F (\nu_f)} = \kappa_S, \tag{10} \]

\[ b [1-F(\nu_f)] = sG(\bar{c}_f), \tag{11} \]

where

\[ W_{Bf} (v) = \frac{\alpha_B \ell_B (\zeta_f)}{r + \alpha_B \ell_B (\zeta_f)} (v - \nu_f), \quad W_{Sf} (c) = \frac{\alpha_S \ell_S (\zeta_f)}{r + \alpha_S \ell_S (\zeta_f)} (\bar{c}_f - c). \]

MW (in Proposition 1) show that, for \( r > 0 \) small, there is a unique solution \( (\zeta_f (r), \nu_f (r), \bar{c}_f (r)) \) to (9) - (11) and it has \( \nu_f > \bar{c}_f \). This solution characterizes a unique equilibrium. The total utility flow of the arriving buyers and sellers, \( W_f (r) \), is obtained in parallel to (8).

### 3 How Private Information Can Be Good For Efficiency

When \( r = 0 \), the buyer-seller ratio and the marginal participating types in full trade equilibria of both models are equal: \( \zeta_f (0) = \zeta_p, \nu_f (0) = \nu_p \) and \( \bar{c}_f (0) = \bar{c}_p \). In a private-information full-trade equilibrium, \( \nu_p \) and \( \bar{c}_p \) do not depend on \( r \). Let

\[ W_{B0} = b \int_0^1 (v - \nu_p) dF (v), \quad W_{S0} = s \int_0^{\zeta_p} (\bar{c}_p - c) dG (c) \]

be the total search values of the arriving flows of buyers and sellers when \( r = 0 \). When there is no discounting, they are the same in both models. Therefore, in order to compare welfare under full and private information for small \( r \), it is sufficient to compare the marginal changes in the welfare, \( dW_p \) and \( dW_f \), as a result of the marginal change in the discount rate \( dr > 0 \). Direct calculations show that

\[ dW_p = - \left[ \frac{W_{B0}}{\ell_B (\zeta_p)} + \frac{W_{S0}}{\ell_S (\zeta_p)} \right] dr, \tag{12} \]

\[ dW_f = - \left[ \frac{1}{\alpha_B} \frac{W_{B0}}{\ell_B (\zeta_p)} + \frac{1}{\alpha_S} \frac{W_{S0}}{\ell_S (\zeta_p)} \right] dr + sG(\bar{c}_p) (d\bar{c}_f - d\nu_f). \tag{13} \]
Under full information, the marginal types are closer to each other than they are under private information. Implicitly differentiating the system (9)-(11) and eliminating \(d\sigma_f, d\sigma_f\) we obtain

\[
d\bar{c_f} - d\bar{\sigma_f} = \frac{dr}{sG(\bar{c}_p)} \left[ \sigma_S \frac{W_{B0}}{\alpha_B \ell_B(\zeta_p)} + \sigma_B \frac{W_{S0}}{\alpha_S \ell_S(\zeta_p)} \right] > 0. \tag{14}
\]

where \(\sigma_S(\zeta) \equiv SM_2(B, S)/M(B, S), \sigma_B(\zeta) \equiv BM_1(B, S)/M(B, S) = 1 - \sigma_S\) are the elasticities of the matching function with respect to the masses of sellers and buyers.

In both models, the marginal entrants get zero rent from bargaining when they respond. Under full information, the marginal entrants extract full rents from the partners when they propose. In contrast, under private information, the marginal entrants are only able to extract the rents of the most inefficient partner type when they propose. If \(r = 0\), this makes no difference because there is no heterogeneity in the partners’ reservation prices. When \(r > 0\), the distributions of reservation prices becomes heterogeneous, and there are rents to be extracted under full information. Of course, the marginal entrants have to be indifferent between entering or not. Hence under full information less efficient types of traders enter.

Substituting \(d\bar{c_f} - d\bar{\sigma_f}\) from (14), we obtain after some algebra

\[
dW_f = dW_p - sG(\bar{c}_p) K'(\zeta_p) d\zeta_f. \tag{15}
\]

This decomposition is intuitive. Under private information, increasing the discount rate from 0 to a small \(dr > 0\) leads to welfare loss given by the r.h.s. of (12) because buyers and sellers in the arriving flow have expected search times until the next profitable trade given by \(1/\ell_B(\zeta_p)\) and \(1/\ell_S(\zeta_p)\), and have their utilities discounted proportionately over those time periods. But under full information, there is also an indirect entry effect of discounting: (14) implies that the full information model has more entry. The new entrants congest the market for their own side, and provide a thicker market for the other side. The entry of traders could increase or decrease the buyer-seller ratio \(\zeta_f\), which in turn affects the expected searching times. The r.h.s. of (15) is the indirect effect on the total accumulated search costs incurred by the flow of buyers and sellers.

**Proposition.** We have

\[
\begin{align*}
\text{Sign } K'(\zeta_p) &= \text{Sign } (\sigma_S(\zeta_p) - \alpha_S), \\
\text{Sign } d\zeta_f &= \text{Sign } \left( \frac{W_{S0}}{\kappa_S} - \frac{W_{B0}}{\kappa_B} \right).
\end{align*}
\]

Therefore, for all sufficiently small \(r > 0\), the private information welfare \(W_p(r)\) is higher (resp. lower) than the full information welfare \(W_f(r)\), if the signs on the r.h.s. are the same (resp., different).

For a concrete example, consider the Cobb-Douglas matching technology, \(M(B, S) = B^{\sigma_B} S^{\sigma_S}\), where \(\sigma_S \in (0, 1), \sigma_B = 1 - \sigma_S\) are constants. Under this matching technology, private information can increase welfare if \(\sigma_S > \alpha_S\) and \(\frac{W_{S0}}{\kappa_S} > \frac{W_{B0}}{\kappa_B}\).

To gain more intuition for our main result, consider the total cost of search \(K(\zeta)\) incurred by an incoming pair of buyer and seller. In Appendix, we show that its derivative \(K'(\zeta)\) has

---

2When \(r = 0\), \(v - W_B(v)\) and \(c + W_S(c)\) are constants in both models.

3See also Lemma 1 in Chapter 3 of Wong (2009).
the same sign as \( \xi - \frac{\sigma_B(\xi)}{\sigma_S(\xi)} \kappa_S \). If \( K(\xi) \) has a unique minimum, as it does in the case of Cobb-
Douglas matching technology above, \( K(\xi) \) is minimized at \( \xi_* \) satisfying \( \xi_* = \frac{\sigma_B(\xi_*)}{\sigma_S(\xi_*)} \kappa_S \). Note
that \( \xi_* = \xi_p \) when the Hosios condition is satisfied: \( \sigma_S(\xi_p) = \alpha_S \). If say \( \xi_p > \xi_* \), \( K'(\xi_p) > 0 \),
and the search cost increases in response to a marginal change \( d\xi > 0 \). Thus, whether the
welfare is higher or lower under full information depends on whether the market tightness
\( \zeta_f \) moves closer or farther away from the optimal level \( \xi_* \) in response to the marginal change
in the discount rate \( dr \).

Appendix: Proof of the Proposition

Since \( K(\xi) = \frac{(\xi \kappa_B + \kappa_S)}{M(\xi, 1)} \), we have

\[
K'(\xi) = \frac{1}{M(\xi, 1)^2} \left[ \kappa_B M(\xi, 1) - M_1(\xi, 1)(\xi \kappa_B + \kappa_S) \right].
\]

Using \( \sigma_B = BM_1(B, S)/M(B, S) \), which is also equal to \( 1 - \sigma_S \) under constant returns to
scale, the above expression can be manipulated to the form

\[
K'(\xi) = \frac{\kappa_B \sigma_S(\xi)}{\xi M(\xi, 1)} \left[ \frac{\xi - \sigma_B(\xi) \kappa_S}{\sigma_S(\xi) \kappa_B} \right].
\]

Therefore the sign of \( K'(\xi) \) is the same as that of \( \xi - \frac{\sigma_B(\xi)}{\sigma_S(\xi)} \kappa_S \). As \( \xi_p = \frac{\alpha_B}{\alpha_S} \kappa_B \), \( K'(\xi_p) \) has
the same sign as either \( \alpha_B - \sigma_B(\xi_p) \) or \( \sigma_S(\xi) - \alpha_S \).

To derive the sign of \( d\xi_f \), divide the buyers' marginal type equation (9) through by
\( \ell_B(\xi_f) \), apply integration by parts to the integral in the left-hand side, differentiate through
at \( r = 0 \), and rearrange. One then obtains

\[
\alpha_B \left[ d\xi_f - d\xi_f + \frac{W_{S0}}{s \alpha_S \ell_S(\xi_p) G(\xi_p)} dr \right] = -\kappa_B \frac{\ell_B(\xi_p)}{\ell_B(\xi_p)} d\xi_f.
\]

Working with the sellers' marginal type equation (10) in the same fashion, we have

\[
\alpha_S \left[ d\xi_f - d\xi_f + \frac{W_{B0}}{b \alpha_B \ell_B(\xi_p) [1 - F(\xi_p)]} dr \right] = -\kappa_S \frac{\ell_S(\xi_p)}{\ell_S(\xi_p)} d\xi_f.
\]

Equations (16) and (17) can be solved for \( d\xi_f - d\xi_f \) and \( d\xi_f \). After some rewriting, we get

\[
d\xi_f = \frac{1}{s G(\xi_p)} \left[ \frac{\sigma_S W_{S0}}{\alpha_B \ell_B(\xi_p)} + \frac{\sigma_B W_{B0}}{\alpha_S \ell_S(\xi_p)} \right] dr,
\]

\[
d\xi_f = \frac{K(\xi_p)}{s G(\xi_p)} \left[ \frac{\kappa_S \ell_S(\xi_p)}{\alpha_S \ell_S(\xi_p)^2} - \frac{\kappa_B \ell_B(\xi_p)}{\alpha_B \ell_B(\xi_p)^2} \right]^{-1} \left( \frac{W_{S0}}{\kappa_S} - \frac{W_{B0}}{\kappa_B} \right) dr.
\]

As \( \ell_S(\xi) > 0 \) and \( \ell_B(\xi) < 0 \), it follows that the sign of \( d\xi_f \) is the same as that of \( \frac{W_{S0}}{\kappa_S} - \frac{W_{B0}}{\kappa_B} \).
References


